YTU – Faculty of Arts and Sciences Exam Questions and Solutions Sheet			Score Table					
			1. q	2. q	3. q	4.q		TOTAL
Name-Surname								
Number		Group No						
Department				Date		08.11. 2019		
Course	MAT1320 Linear Algebra		Duration	80 min.	Room			
Lecturer				Signature				

YÖK nun 2547 sayılı Kanunun *Öğrenci Disiplin Yönetmeliğinin* 9. Maddesi olan *"Sınavlarda kopya yapmak ve yaptırmak veya buna teşebbüs etmek"* fiili işleyenler bir veya iki yarıyıl uzaklaştırma cezası alırlar.

1) Using elementary row operations, find the inverse of the matrix $A=\begin{pmatrix} 1 & 0 & 2 \\ -1 & 0 & 1 \\ 2 & 3 & 0 \end{pmatrix}$.

$$\left[\begin{array}{ccccccc} 1 & 0 & 2 & : & 1 & 0 & 0 \\ -1 & 0 & 1 & : & 0 & 1 & 0 \\ 2 & 3 & 0 & : & 0 & 0 & 1 \end{array}\right] \xrightarrow{r_2 \leftrightarrow r_3} \left[\begin{array}{ccccccc} 1 & 0 & 2 & : & 1 & 0 & 0 \\ 2 & 3 & 0 & : & 0 & 0 & 1 \\ -1 & 0 & 1 & : & 0 & 1 & 0 \end{array}\right] \xrightarrow[r_3 \to r_3 + r_1]{r_2 \to r_2 - 2r_1}$$

$$\left[\begin{array}{ccccccc} 1 & 0 & 2 & : & 1 & 0 & 0 \\ 0 & 3 & -4 & : & -2 & 0 & 1 \\ 0 & 0 & 3 & : & 1 & 1 & 0 \end{array}\right] \xrightarrow{r_3 \to \frac{1}{3} \cdot r_3} \left[\begin{array}{ccccccc} 1 & 0 & 2 & : & 1 & 0 & 0 \\ 0 & 3 & -4 & : & -2 & 0 & 1 \\ 0 & 0 & 1 & : & \frac{1}{3} & \frac{1}{3} & 0 \end{array}\right] \xrightarrow[r_1 \to r_1 - 2r_3]{r_2 \to r_2 + 4r_3}$$

$$\left[\begin{array}{ccccccc} 1 & 0 & 0 & : & \frac{1}{3} & \frac{-2}{3} & 0 \\ 0 & 3 & 0 & : & \frac{-2}{3} & \frac{4}{3} & 1 \\ 0 & 0 & 1 & : & \frac{1}{3} & \frac{1}{3} & 0 \end{array}\right] \xrightarrow{r_2 \to \frac{1}{3} \cdot r_2} \left[\begin{array}{ccccccc} 1 & 0 & 0 & : & \frac{1}{3} & \frac{-2}{3} & 0 \\ 0 & 1 & 0 & : & \frac{-2}{9} & \frac{4}{9} & \frac{1}{3} \\ 0 & 0 & 1 & : & \frac{1}{3} & \frac{1}{3} & 0 \end{array}\right]$$

$$\Rightarrow A^{-1} = \begin{bmatrix} \frac{1}{3} & \frac{-2}{3} & 0 \\ \frac{-2}{9} & \frac{4}{9} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & 0 \end{bmatrix}$$

Good Luck...

2) $n \times n$ tipinde bir $A = (a_{ij})$ matrisinin izi $Tr(A)$ ile gösterilsin.

a) $Tr(A^T A) \geq 0$ olduğunu gösteriniz. **(13 Puan)**

$A^T A = B = (b_{ij})$ olsun.

$$b_{ii} = \sum_{j=1}^{n} a^T_{ij} \cdot a_{ji} = \sum_{j=1}^{n} a_{ji}^2 \Longrightarrow$$

$$Tr(B) = Tr(A^T A) = \sum_{i=1}^{n} b_{ii}$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}^2 \geq 0$$

b) $AB - BA = I_2$ eşitliğini sağlayan 2×2 tipinde A ve B matrislerinin var olmadığını gösteriniz. **(12 Puan)**

$$Tr(AB - BA) = Tr(AB) - Tr(BA) = 0$$

iken $Tr(I_2) = 2$.

Başarılar…

3) Suppose that $f:\mathbb{R}\to M_2(\mathbb{R})$ is a function from the set of real numbers to the set $M_2(\mathbb{R})$ of 2×2 matrices such that $f(x)=\begin{pmatrix}\cos x & -\sin x\\ \sin x & \cos x\end{pmatrix}$.

a) Show that $f(x+y)=f(x)f(y)$ for all $x,y\in\mathbb{R}$.

$$f(x)f(y)=\begin{pmatrix}\cos x & -\sin x\\ \sin x & \cos x\end{pmatrix}\begin{pmatrix}\cos y & -\sin y\\ \sin y & \cos y\end{pmatrix}$$

$$=\begin{pmatrix}\cos x\cos y-\sin x\sin y & -\cos x\sin y-\sin x\cos y\\ \sin x\cos y+\cos x\sin y & -\sin x\sin y+\cos x\cos y\end{pmatrix}$$

$$=\begin{pmatrix}\cos(x+y) & -\sin(x+y)\\ \sin(x+y) & \cos(x+y)\end{pmatrix}=f(x+y)$$

b) Show that $f(x)$ is an orthogonal matrix for all x in $\mathbb{R}$.

$f(x)\cdot(f(x))^T=I_2$ olmalı.

$$\begin{pmatrix}\cos x & -\sin x\\ \sin x & \cos x\end{pmatrix}\begin{pmatrix}\cos x & \sin x\\ -\sin x & \cos x\end{pmatrix}=\begin{pmatrix}\cos^2 x+\sin^2 x & \cos x\sin x-\sin x\cos x\\ \sin x\cos x-\sin x\cos x & \sin^2 x+\cos^2 x\end{pmatrix}$$

$$=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

$\Rightarrow$ $f(x)$ matrisi ortogonal.

Good Luck...

4) $A = \begin{pmatrix} 2 & 1 & 3 \\ -1 & 2 & 0 \\ 3 & -2 & 1 \end{pmatrix}$ matrisi verilsin.

a) $Ek(A)$ ve $\det(A)$'yı hesaplayınız. **(13 Puan)**

$$Ek(A) = \begin{pmatrix} \begin{vmatrix} 2 & 0 \\ -2 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ -2 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 2 & 0 \end{vmatrix} \\ -\begin{vmatrix} -1 & 0 \\ 3 & 1 \end{vmatrix} & \begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} & -\begin{vmatrix} 2 & 3 \\ -1 & 0 \end{vmatrix} \\ \begin{vmatrix} -1 & 2 \\ 3 & -2 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ 3 & -2 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix} \end{pmatrix} = \begin{pmatrix} 2 & -7 & -6 \\ 1 & -7 & -3 \\ \cancel{4} & 7 & 5 \end{pmatrix}$$

$$\det(A) = 3 \cdot \det\begin{pmatrix} -1 & 2 \\ 3 & -2 \end{pmatrix} - 0 \cdot \det\begin{pmatrix} 2 & 1 \\ 3 & -2 \end{pmatrix} + 1 \cdot \det\begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix}$$

$$= 3(-4) + 5 = -12 + 5 = -7$$

b) $Ek(A)A = \det(A)I_3$ eşitliğinin doğru olduğunu gösteriniz. **(12 Puan)**

$$Ek(A)A = \begin{pmatrix} 2 & -7 & -6 \\ 1 & -7 & -3 \\ -4 & 7 & 5 \end{pmatrix} \begin{pmatrix} 2 & 1 & 3 \\ -1 & 2 & 0 \\ 3 & -2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -7 & 0 & 0 \\ 0 & -7 & 0 \\ 0 & 0 & -7 \end{pmatrix} = -7 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \det(A) \cdot I_3$$

Başarılar…