

## SINAV SORULARI GÖZÜMLERİ

$$1) \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin x - 1}{\cos^2 x} = \frac{0}{0} \text{ bl.}$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin x - 1}{\frac{\cos^2 x}{\sin^2 x}} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{(\sin x - 1) \sin^2 x}{\cos^2 x} \cdot \frac{(\sin x + 1)}{(\sin x + 1)}$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \frac{(\sin^2 x - 1) \sin^2 x}{\cos^2 x (\sin x + 1)} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{-\cancel{\cos^2 x} \cdot \sin^2 x}{\cancel{\cos^2 x} (\sin x + 1)} = -\frac{1}{2} //$$

$$2) \lim_{x \rightarrow 0^+} x^{\frac{1}{\ln(x-1)}} = 0^\infty \text{ bl.}$$

$$y = x^{\frac{1}{\ln(x-1)}}$$

$$\ln y = \frac{1}{\ln(x-1)} \ln x$$

$$\Rightarrow \lim_{x \rightarrow 0^+} \ln y = \lim_{x \rightarrow 0^+} \frac{\ln x}{\ln(x-1)} \left( \frac{\infty}{\infty} \text{ bl.} \right)$$

$$\stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0^+} \frac{\frac{1}{x}}{\frac{e^x}{e^x - 1}} = \lim_{x \rightarrow 0^+} \frac{e^x - 1}{x e^x} = \frac{0}{0} \text{ bl.}$$

$$\stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0^+} \frac{e^x}{e^x + x e^x} = 1$$

$$\lim_{x \rightarrow 0^+} \ln y = \ln \lim_{x \rightarrow 0^+} y = 1$$

$$\Rightarrow \lim_{x \rightarrow 0^+} x^{\frac{1}{\ln(x-1)}} = e^1 = e //$$

$$3) \lim_{x \rightarrow \pi} \frac{\sin(2x)}{x^2 - \pi x} = \frac{0}{0} \text{ bl.}$$

$$= \lim_{x \rightarrow \pi} \frac{2 \cdot \sin x \cdot \cos x}{x(x - \pi)}$$

$$= \lim_{t \rightarrow 0} \frac{2 \cdot \sin(\pi + t) \cdot \cos(\pi + t)}{(\pi + t) \cdot t}$$

$$= \lim_{t \rightarrow 0} \frac{2 \cdot (-\sin t) \cdot (-\cos t)}{t \cdot (\pi + t)} = \lim_{t \rightarrow 0} \frac{\sin t}{t} \cdot \frac{2 \cos t}{(\pi + t)} = \frac{2}{\pi} //$$

$$x - \pi = t \Rightarrow x = t + \pi$$

$$x \rightarrow \pi \Rightarrow t = 0$$

$$\sin(\pi + t) = -\sin t$$

$$\cos(\pi + t) = -\cos t$$



$$\begin{aligned}
 7) \quad & \lim_{x \rightarrow \infty} \frac{x - \sqrt{1+x^2}}{x - \sqrt{x}} \quad \left( \frac{\infty}{\infty} \text{ bl.} \right) \\
 & = \lim_{x \rightarrow \infty} \frac{x - \sqrt{x^2 \left( \frac{1}{x^2} + 1 \right)}}{x \left( 1 - \frac{1}{\sqrt{x}} \right)} = \lim_{x \rightarrow \infty} \frac{x - |x| \sqrt{\frac{1}{x^2} + 1}}{x \left( 1 - \frac{1}{\sqrt{x}} \right)} \\
 & = \lim_{x \rightarrow \infty} \frac{\cancel{x} \left( 1 - \sqrt{\frac{1}{x^2} + 1} \right)}{\cancel{x} \left( 1 - \frac{1}{\sqrt{x}} \right)} = \frac{1-1}{1} = 0 \quad //
 \end{aligned}$$

$$8) \quad \lim_{x \rightarrow 0^+} \left( 2 - e^{\sqrt{x}} \right)^{\frac{1}{x}} \quad (1^\infty \text{ bl.})$$

$$y = \left( 2 - e^{\sqrt{x}} \right)^{\frac{1}{x}}$$

$$\ln y = \frac{1}{x} \ln \left( 2 - e^{\sqrt{x}} \right)$$

$$\lim_{x \rightarrow 0^+} \ln y = \lim_{x \rightarrow 0^+} \frac{\ln \left( 2 - e^{\sqrt{x}} \right)}{x} = \left( \frac{0}{0} \text{ bl.} \right)$$

$$\stackrel{\text{UH}}{=} \lim_{x \rightarrow 0^+} \frac{-\frac{1}{2\sqrt{x}} \cdot e^{\sqrt{x}}}{2 - e^{\sqrt{x}}} = -\infty$$

$$\lim_{x \rightarrow 0^+} \ln y = \ln \left( \lim_{x \rightarrow 0^+} y \right) = -\infty$$

$$\Rightarrow \lim_{x \rightarrow 0^+} \left( 2 - e^{\sqrt{x}} \right)^{\frac{1}{x}} = e^{-\infty} = 0 \quad //$$

$$9) \quad \lim_{x \rightarrow -\infty} \left( 2x + \sqrt{4x^2 + 3x} \right) = (\infty - \infty \text{ bl.})$$

$$= \lim_{x \rightarrow -\infty} \frac{\left( 2x + \sqrt{4x^2 + 3x} \right) \left( 2x - \sqrt{4x^2 + 3x} \right)}{\left( 2x - \sqrt{4x^2 + 3x} \right)}$$

$$= \lim_{x \rightarrow -\infty} \frac{4x^2 - 4x^2 - 3x}{2x - \sqrt{4x^2 + 3x}} = \lim_{x \rightarrow -\infty} \frac{-3x}{2x - |x| \sqrt{4 + \frac{3}{x}}} = \lim_{x \rightarrow -\infty} \frac{-3x}{2x + x \sqrt{4 + \frac{3}{x^2}}}$$

$$= \lim_{x \rightarrow -\infty} \frac{-3x}{x \left( 2 + \sqrt{4 + \frac{3}{x^2}} \right)} = -\frac{3}{4} \quad //$$

$$10) \lim_{x \rightarrow 0^+} (\cot x)^{1/\ln x} \quad (\infty^0 \text{ bl})$$

$$y = (\cot x)^{1/\ln x}$$

$$\ln y = \frac{1}{\ln x} \cdot \ln \cot x$$

$$\lim_{x \rightarrow 0^+} \ln y = \lim_{x \rightarrow 0^+} \frac{\ln \cot x}{\ln x} \stackrel{\infty/\infty \text{ bl}}{=} \lim_{x \rightarrow 0^+} \frac{\frac{\cos x}{\sin x}}{\frac{1}{x}} = \lim_{x \rightarrow 0^+} \frac{-x \cdot \sin x}{\sin^2 x \cdot \cos x}$$

$$= \lim_{x \rightarrow 0^+} \frac{-x}{\sin x \cdot \cos x} = \lim_{x \rightarrow 0^+} \frac{x}{\sin x} \cdot \frac{-1}{\cos x} = -1$$

$$\lim_{x \rightarrow 0^+} \ln y = \ln \left( \lim_{x \rightarrow 0^+} y \right) = -1$$

$$\Rightarrow \lim_{x \rightarrow 0^+} (\cot x)^{1/\ln x} = e^{-1} = \frac{1}{e} //$$

$$11) \lim_{x \rightarrow 0} \frac{1 - \cos(\sin x)}{2x^2} \quad \left( \frac{0}{0} \text{ bl.} \right)$$

$$= \lim_{x \rightarrow 0} \frac{(1 - \cos(\sin x)) \cdot (1 + \cos(\sin x))}{2x^2 (1 + \cos(\sin x))}$$

$$= \lim_{x \rightarrow 0} \frac{1 - \cos^2(\sin x)}{2x^2 (1 + \cos(\sin x))} = \lim_{x \rightarrow 0} \frac{\sin^2(\sin x)}{2x^2 (1 + \cos(\sin x))}$$

$$= \lim_{x \rightarrow 0} \frac{\sin^2(\sin x)}{\sin^2 x} \cdot \frac{\sin^2 x}{x^2} \cdot \frac{1}{2(1 + \cos(\sin x))} = \frac{1}{4} //$$

$$12) \lim_{x \rightarrow \infty} \frac{e^{\arctan x} - x}{\ln(1+x^2) + x} \quad (\infty/\infty \text{ bl}) \stackrel{\text{L'H}}{=} \lim_{x \rightarrow \infty} \frac{\frac{1}{1+x^2} \cdot e^{\arctan x} - 1}{\frac{2x}{1+x^2} + 1}$$

$$= \frac{0 \cdot e^{\pi/2} - 1}{0 + 1} = -1 //$$

$$13) \lim_{x \rightarrow 0^+} \left( \frac{a^x + b^x}{2} \right)^{\frac{2}{x}} \quad (1^\infty \text{ bl.})$$

$$y = \left( \frac{a^x + b^x}{2} \right)^{\frac{2}{x}}$$

$$\ln y = \frac{2}{x} \ln \left( \frac{a^x + b^x}{2} \right)$$

$$\lim_{x \rightarrow 0^+} \ln y = \lim_{x \rightarrow 0^+} \frac{\ln \left( \frac{a^x + b^x}{2} \right)}{\frac{x}{2}} \quad \left( \frac{0}{0} \text{ bl.} \right) \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0^+} \frac{2 \cdot \frac{a^x \ln a + b^x \ln b}{2(a^x + b^x)}}{\frac{1}{2}}$$

$$= \lim_{x \rightarrow 0^+} \frac{2(a^x \ln a + b^x \ln b)}{a^x + b^x} = \ln a + \ln b$$

$$\lim_{x \rightarrow 0^+} \ln y = \ln \left( \lim_{x \rightarrow 0^+} y \right) = \ln(a \cdot b)$$

$$\Rightarrow \lim_{x \rightarrow 0^+} \left( \frac{a^x + b^x}{2} \right)^{\frac{2}{x}} = a \cdot b \quad //$$

$$14) \lim_{x \rightarrow 2} \frac{|x^2 - 4|}{\sqrt[4]{x-2}}$$

$$\lim_{x \rightarrow 2^+} \frac{|x^2 - 4|}{\sqrt[4]{x-2}} = \lim_{x \rightarrow 2^+} \frac{(x-2)(x+2)}{(x-2)^{1/4}} = \lim_{x \rightarrow 2^+} (x-2)^{3/4} (x+2) = 0$$

$$\lim_{x \rightarrow 2^-} \frac{|x^2 - 4|}{\sqrt[4]{x-2}} = \lim_{x \rightarrow 2^-} \frac{-(x-2)(x+2)}{(x-2)^{1/4}} = \lim_{x \rightarrow 2^-} -(x-2)^{3/4} (x+2) = 0$$

$$\lim_{x \rightarrow 2} \frac{|x^2 - 4|}{\sqrt{x-2}} = 0 \quad //$$

$$15) \lim_{x \rightarrow 1^+} \left[ \frac{4}{\pi} \arctan x \right]^{\frac{3}{x^2+2x-3}} \quad (1^\infty \text{ bl.})$$

$$y = \left[ \frac{4}{\pi} \arctan x \right]^{\frac{3}{x^2+2x-3}}$$

$$\ln y = \frac{3}{(x^2+2x-3)} \ln \left( \frac{4}{\pi} \arctan x \right)$$

$$\lim_{x \rightarrow 1^+} \ln y = \lim_{x \rightarrow 1^+} \frac{3 \cdot \ln \left( \frac{4}{\pi} \arctan x \right)}{(x^2+2x-3)} \quad \left( \frac{0}{0} \text{ bl.} \right)$$

$$\stackrel{\text{L'H}}{=} \lim_{x \rightarrow 1^+} \frac{3 \cdot \frac{\frac{4}{\pi} \cdot \frac{1}{1+x^2}}{\frac{4}{\pi} \arctan x}}{2x+2} = \lim_{x \rightarrow 1^+} \frac{3}{(2x+2)(1+x^2) \arctan x}$$

$$\lim_{x \rightarrow 1^+} \ln y = \frac{3}{4 \cdot 2 \cdot \frac{\pi}{4}} = \frac{3}{2\pi}$$

$$\ln \left( \lim_{x \rightarrow 1^+} y \right) = \frac{3}{2\pi} \Rightarrow \lim_{x \rightarrow 1^+} \left[ \frac{4}{\pi} \arctan x \right]^{\frac{3}{x^2+2x-3}} = e^{\frac{3}{2\pi}} //$$

$$16) \lim_{x \rightarrow 0^+} (1 + \sin 2x)^{\frac{1}{x}} \quad (1^\infty \text{ bl.})$$

$$y = (1 + \sin 2x)^{\frac{1}{x}}$$

$$\ln y = \frac{1}{x} \ln(1 + 2x)$$

$$\lim_{x \rightarrow 0^+} \ln y = \lim_{x \rightarrow 0^+} \frac{\ln(1+2x)}{x} \quad \left( \frac{0}{0} \text{ bl.} \right)$$

$$\stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0^+} \frac{\frac{2}{1+2x}}{1} = 2$$

$$\lim_{x \rightarrow 0^+} \ln y = 2$$

$$\ln \left( \lim_{x \rightarrow 0^+} y \right) = 2 \Rightarrow \lim_{x \rightarrow 0^+} (1 + \sin 2x)^{\frac{1}{x}} = e^2 //$$

17)  $f(x) = \ln(\ln(\ln x)) + \sqrt{9-x^2}$

$\ln(\ln(\ln x)) > 0$

$\ln(\ln x) > 1$

$\ln x > e$

$x > e$

$x > 0$

$\ln x > 0$

$x > 1$

$9-x^2 \geq 0$

x	-3	3
9-x <sup>2</sup>	-	+
	[-3, 3]	

T.K. : (e, 3]

18)  $f(x) = \frac{1}{\sqrt{|x|-x}} + \ln\left(\frac{9-x^2}{x^2+x}\right)$

$|x|-x > 0$

$|x| > x$

$x < 0$

$(-\infty, 0)$

$\frac{9-x^2}{x^2+x} > 0$

$\frac{(3-x)(3+x)}{x(x+1)} > 0$

x	-3	-1	0	3
9-x <sup>2</sup>	-	+	+	-
x	-	-	+	+
x+1	-	-	+	+
$\frac{9-x^2}{x(x+1)}$	-	+	-	+

$(-3, -1) \cup (0, 3)$

$(-\infty, 0) \cap ((-3, -1) \cup (0, 3))$

T.K. : (-3, -1)

//

$$19) f(x) = \begin{cases} \frac{\pi}{2} & , x=0 \\ \sin\left(\frac{x}{3}\right) & , 0 < x < 3 \\ 2^{\frac{1}{x-4}} & , 3 \leq x < 4 \text{ ve } 4 < x \leq 5 \\ \frac{\pi}{2} & , x=4 \end{cases}$$

$$\begin{array}{l} \underline{x=0} \\ \lim_{x \rightarrow 0^+} \sin\left(\frac{x}{3}\right) = 0 \\ f(0) = \frac{\pi}{2} \end{array} \left. \vphantom{\begin{array}{l} \lim_{x \rightarrow 0^+} \sin\left(\frac{x}{3}\right) = 0 \\ f(0) = \frac{\pi}{2} \end{array}} \right\} \neq x=0 \text{ da kaldırılabilir süreksizdir.}$$

$$\begin{array}{l} \underline{x=3} \\ \lim_{x \rightarrow 3^+} 2^{\frac{1}{x-4}} = \frac{1}{2} = f(3) \\ \lim_{x \rightarrow 3^-} \sin\left(\frac{x}{3}\right) = \sin 1 \end{array} \left. \vphantom{\begin{array}{l} \lim_{x \rightarrow 3^+} 2^{\frac{1}{x-4}} = \frac{1}{2} = f(3) \\ \lim_{x \rightarrow 3^-} \sin\left(\frac{x}{3}\right) = \sin 1 \end{array}} \right\} \neq x=3 \text{ de sıçramalı süreksizdir.}$$

$$\begin{array}{l} \underline{x=4} \\ \lim_{x \rightarrow 4^+} 2^{\frac{1}{x-4}} = 2^{\infty} = \infty \\ \lim_{x \rightarrow 4^-} 2^{\frac{1}{x-4}} = 2^{-\infty} = 0 \end{array} \left. \vphantom{\begin{array}{l} \lim_{x \rightarrow 4^+} 2^{\frac{1}{x-4}} = 2^{\infty} = \infty \\ \lim_{x \rightarrow 4^-} 2^{\frac{1}{x-4}} = 2^{-\infty} = 0 \end{array}} \right\} x=4 \text{ de sonsuz (esas) süreksizdir.}$$



$$20) f(x) = \begin{cases} x & , x \leq -2 \\ 1/(x+2) & , -2 < x \leq 1 \\ \frac{\sin(1-\sqrt{x})}{x-1} & , x > 1 \end{cases}$$

i)  $\lim_{x \rightarrow -2} f(x)$  ,  $\lim_{x \rightarrow 1} f(x)$

$$\lim_{x \rightarrow -2^-} x = -2$$

$$\lim_{x \rightarrow -2^+} \frac{1}{x+2} = \infty$$

}  $\lim_{x \rightarrow -2} f(x)$  mevcut değildir.

$$\lim_{x \rightarrow 1^-} \frac{1}{x+2} = \frac{1}{3}$$

$$\lim_{x \rightarrow 1^+} \frac{\sin(1-\sqrt{x})}{x-1} = \lim_{x \rightarrow 1^+} \frac{\sin(1-\sqrt{x})}{(\sqrt{x}-1)(\sqrt{x}+1)}$$

$$= \lim_{x \rightarrow 1^+} \frac{\sin(1-\sqrt{x})}{\underbrace{(1-\sqrt{x})}_1} \cdot \frac{-1}{(\sqrt{x}+1)} = -\frac{1}{2}$$

}  $\neq$  olduğundan  $\lim_{x \rightarrow 1} f(x)$  mevcut değildir.

ii)  $\lim_{x \rightarrow -2^+} \frac{1}{x+2} = \infty$  olduğundan  $x = -2$  de sonsuz (esas) süreksiz

$$\lim_{x \rightarrow 1^-} \frac{1}{x+2} = \frac{1}{3} = f(1)$$

$$\lim_{x \rightarrow 1^+} \frac{\sin(1-\sqrt{x})}{x-1} = -\frac{1}{2}$$

}  $\neq$  olduğundan

$x = 1$  de  $f(x)$  sıçramalı süreksiz

$$21) f(x) = \frac{3|x-2|}{x^2(4-x^2)}$$

$x=0$  ,  $x = \pm 2$  süreksizlik noktaları

$$\lim_{x \rightarrow 0} \frac{3|x-2|}{x^2(2-x)(2+x)} = \lim_{x \rightarrow 0} \frac{3(2-x)}{x^2(2-x)(2+x)} = \infty \quad \text{sonsuz (esas) süreksizdir.}$$

$x=2$

$$\lim_{x \rightarrow 2^+} \frac{3|x-2|}{x^2(2-x)(2+x)} = \lim_{x \rightarrow 2^+} \frac{3(x-2)}{x^2(2-x)(2+x)} = \frac{-3}{16}$$

$$\lim_{x \rightarrow 2^-} \frac{3|x-2|}{x^2(2-x)(2+x)} = \lim_{x \rightarrow 2^-} \frac{3(2-x)}{x^2(2-x)(2+x)} = \frac{3}{16}$$

}  $\neq$  sıçramalı süreksizdir.

$x=-2$

$$\lim_{x \rightarrow -2^+} \frac{3|x-2|}{x^2(2-x)(2+x)} = \lim_{x \rightarrow -2^+} \frac{3(2-x)}{x^2(2-x)(2+x)} = \infty \quad \text{sonsuz (esas) süreksiz}$$

$$22) f(x) = \frac{2|x-1|}{x^2-x^3}$$

$x^2(1-x) = 0 \Rightarrow x=0$  ,  $x=1$  süreksizlik noktaları

$$\lim_{x \rightarrow 0} \frac{2|x-1|}{x^2(1-x)} = \lim_{x \rightarrow 0} \frac{2(1-x)}{x^2(1-x)} = \infty \quad \text{sonsuz (esas) süreksizlik}$$

$x=1$

$$\lim_{x \rightarrow 1^+} \frac{2|x-1|}{x^2(1-x)} = \lim_{x \rightarrow 1^+} \frac{2(x-1)}{x^2(1-x)} = \frac{-2}{1} = -2$$

$$\lim_{x \rightarrow 1^-} \frac{2|x-1|}{x^2(1-x)} = \lim_{x \rightarrow 1^-} \frac{2(1-x)}{x^2(1-x)} = \frac{2}{1} = 2$$

}  $\neq$  sıçramalı süreksizlik

23)  $f(x) = \frac{x^2}{1-\cos x}$   $x=0$  da fonksiyon tanımsız.

$$\lim_{x \rightarrow 0} \frac{x^2}{1-\cos x} = \lim_{x \rightarrow 0} \frac{x^2 (1+\cos x)}{1-\cos^2 x} = \lim_{x \rightarrow 0} \frac{x^2}{\frac{\sin^2 x}{1}} (1+\cos x) = 2.$$

$x=0$  da  $f(x)$  kaldırılabilir süreksizliğe sahiptir.

$$F(x) = \begin{cases} \frac{x^2}{1-\cos x} & , x \neq 0 \\ 2 & , x = 0 \end{cases}$$

24)  $f(x) = \frac{|x^2-9|}{x^2-4x+3}$   
 $x^2-4x+3=0 \Rightarrow (x-3)(x-1)=0 \Rightarrow x=1, x=3$  süreksizlik noktaları

$x=1$   
 $\lim_{x \rightarrow 1^+} \frac{|x^2-9|}{(x-1)(x-3)} = \lim_{x \rightarrow 1^+} \frac{-(x-3)(x+3)}{(x-1)(x/3)} = -\infty$  sonsuz (esas) süreksizlik

$x=3$   
 $\lim_{x \rightarrow 3^+} \frac{|x^2-9|}{(x-1)(x-3)} = \lim_{x \rightarrow 3^+} \frac{(x-3)(x+3)}{(x-1)(x/3)} = \frac{6}{2} = 3$   
 $\lim_{x \rightarrow 3^-} \frac{|x^2-9|}{(x-1)(x-3)} = \lim_{x \rightarrow 3^-} \frac{-(x-3)(x+3)}{(x-1)(x/3)} = -3$   
 $\neq$  sıçramalı süreksizlik

25)

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x^2} & , x < 0 \\ a - \arcsin\left(\frac{x+1}{2}\right) & , 0 \leq x < 1 \\ \frac{a}{b} + \arctan \sqrt{3} x & , x \geq 1 \end{cases}$$

$$\lim_{x \rightarrow 0^-} x^2 \cdot \frac{\sin \frac{1}{x^2}}{x^2} = 0$$

$$\lim_{x \rightarrow 0^+} a - \arcsin\left(\frac{x+1}{2}\right) = a - \frac{\pi}{6} = f(0)$$

$$\Rightarrow a - \frac{\pi}{6} = 0 \Rightarrow \boxed{a = \frac{\pi}{6}}$$

$$\lim_{x \rightarrow 1^-} \left( a - \arcsin\left(\frac{x+1}{2}\right) \right) = a - \frac{\pi}{2} = \frac{\pi}{6} - \frac{\pi}{2} = -\frac{\pi}{3}$$

$$\lim_{x \rightarrow 1^+} \left( \frac{a}{b} + \arctan \sqrt{3} x \right) = \frac{a}{b} + \frac{\pi}{3} = f(1)$$

$$\frac{\pi/6}{b} + \frac{\pi}{3} = -\frac{\pi}{3}$$

$$\boxed{b = -\frac{1}{4}}$$

26)

Ara Değer Teoremi: Eğer  $f$   $[a, b]$  aralığında sürekli bir fonksiyon ve  $y_0$  da  $f(a)$  ve  $f(b)$  arasında herhangi bir değer ise bu durumda  $y_0 = f(c)$  olacak şekilde  $(a, b)$  aralığında bazı  $c$ 'ler vardır.

$f(x) = x - \cos x$  olsun.  $f$  fonksiyonu  $[0, \frac{\pi}{2}]$  aralığında sürekli,  $f(0) = -1$  ve  $f(\frac{\pi}{2}) = \frac{\pi}{2}$  olduğundan, Ara Değer Teoremine göre,

$$f(0) = -1 < f(c) = 0 < f\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$$

$f(c) = 0 = c - \cos c$  olacak şekilde bir  $c \in [0, \frac{\pi}{2}]$  vardır.

$$\cos c = c //$$

$$27) \quad x > 0, \quad f(x) = \frac{1}{\sqrt{x}}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{x} - \sqrt{x+h}}{h \cdot \sqrt{x}\sqrt{x+h}} \cdot \frac{(\sqrt{x} + \sqrt{x+h})}{(\sqrt{x} + \sqrt{x+h})}$$

$$= \lim_{h \rightarrow 0} \frac{x - (x+h)}{h \sqrt{x} \cdot \sqrt{x+h} (\sqrt{x} + \sqrt{x+h})} = \lim_{h \rightarrow 0} \frac{-h}{h \sqrt{x} \sqrt{x+h} (\sqrt{x} + \sqrt{x+h})} \cdot \frac{1}{(\sqrt{x} + \sqrt{x+h})}$$

$$= -\frac{1}{2x\sqrt{x}} = -\frac{1}{2\sqrt{x^3}} \quad //$$

$$28) \quad 0 < x < 1, \quad f(x) = \arcsin x - \arccos \sqrt{1-x^2}$$

$$f'(x) = \frac{1}{\sqrt{1-x^2}} - \frac{\frac{2x}{2\sqrt{1-x^2}}}{\sqrt{1-(1-x^2)}} = \frac{1}{\sqrt{1-x^2}} - \frac{x}{x\sqrt{1-x^2}} = \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} = 0$$

$f'(x) = 0 \Rightarrow f(x) = C$  şeklinde sabit bir fonksiyondur.

$$29) \quad f(x) = \begin{cases} \sqrt[3]{x} (1 - \cos x), & x \geq 0 \\ \sin x, & x < 0 \end{cases}$$

$$\left. \begin{array}{l} \lim_{x \rightarrow 0^+} \sqrt[3]{x} (1 - \cos x) = 0 = f(0) \\ \lim_{x \rightarrow 0^-} \sin x = 0 \end{array} \right\} \Rightarrow f(x), x=0 \text{ da s\u00fcreklidir.}$$

$$f'_+(0) = \lim_{h \rightarrow 0^+} \frac{f(0+h) - f(0)}{h} = \lim_{h \rightarrow 0^+} \frac{\sqrt[3]{h} (1 - \cos h)}{h} = 0$$

$$f'_-(0) = \lim_{h \rightarrow 0^-} \frac{f(0+h) - f(0)}{h} = \lim_{h \rightarrow 0^-} \frac{\sin h}{h} = 1$$

$f'_+(0) \neq f'_-(0)$  olduğundan  $f(x)$ ,  $x=0$  da türevlenemezdir.

$$30) f(g(x)) = x \quad f'(x) = 1 + (f(x))^2$$

$$(f(g(x)))' = 1$$

$$g'(x) \cdot f'(g(x)) = 1$$

$$g'(x) = \frac{1}{f'(g(x))} = \frac{1}{1 + (f(g(x)))^2} = \frac{1}{1 + x^2} //$$

$$31) \lim_{h \rightarrow 1} \frac{f(h) - f(2-h)}{h-1}$$

$$h = t+1$$

$$h \rightarrow 1 \Rightarrow t \rightarrow 0$$

$$= \lim_{t \rightarrow 0} \frac{f(t+1) - f(1-t) + f(1) - f(1)}{t}$$

$$= \lim_{t \rightarrow 0} \frac{f(1+t) - f(1)}{t} + \lim_{t \rightarrow 0} \frac{f(1) - f(1-t)}{t}$$

$$= f'(1) + \lim_{t \rightarrow 0} \frac{f(1-t) - f(1)}{-t}$$

$$\lim_{h \rightarrow 1} \frac{f(h) - f(2-h)}{h-1} = f'(1) + f'(1) = 2f'(1) //$$

$$32) f(x) = \begin{cases} x^2 + x, & x < 0 \\ x + 1, & x \geq 0 \end{cases}$$

$x=0$  da sürekli mi?

$$\lim_{x \rightarrow 0^+} x + 1 = 1 = f(0)$$

$$\lim_{x \rightarrow 0^-} x^2 + x = 0$$

$\neq$  olduğundan  $f(x)$ ,  $x=0$  da sürekli değildir.

Dolayısıyla  $x=0$  da türevi mevcut değildir.

$$33) f(x) = \begin{cases} \tan(\sin x) & , x \leq 0 \\ \frac{1}{x} \sin x^2 & , x > 0 \end{cases}$$

$$\lim_{x \rightarrow 0^+} \frac{1}{x} \sin x^2 = \lim_{x \rightarrow 0^+} \frac{\sin x^2}{\underbrace{x^2}_1} \cdot x = 0$$

$$\lim_{x \rightarrow 0^-} \tan(\sin x) = 0 = f(0)$$

lim  $f(x) = f(0)$  olduğundan  $x=0$  da süreklidir.

$$f'_+(0) = \lim_{h \rightarrow 0^+} \frac{f(0+h) - f(0)}{h} = \lim_{h \rightarrow 0^+} \frac{\frac{1}{h} \sin h^2}{h} = \lim_{h \rightarrow 0^+} \frac{\sin h^2}{h^2} = 1$$

$$f'_-(0) = \lim_{h \rightarrow 0^-} \frac{f(0+h) - f(0)}{h} = \lim_{h \rightarrow 0^-} \frac{\tan(\sin h)}{h} = \lim_{h \rightarrow 0^-} \frac{\tan(\sin h)}{\sin h} \cdot \frac{\sin h}{h} = 1$$

$f'_+(0) = f'_-(0) = 1$  olduğundan  $f(x)$ ,  $x=0$  de türemlenebilir.

$$f'(0) = 1 //$$

$$34) f(x) = \begin{cases} x+1 & , x < 0 \\ \cos^2 x & , x \geq 0 \end{cases}$$

$$\lim_{x \rightarrow 0^+} \cos^2 x = 1 = f(0)$$

$$\lim_{x \rightarrow 0^-} x+1 = 1$$

olduğundan  $x=0$  da süreklidir.

$$f'_+(0) = \lim_{h \rightarrow 0^+} \frac{f(0+h) - f(0)}{h} = \lim_{h \rightarrow 0^+} \frac{\cos^2 h - 1}{h} = \lim_{h \rightarrow 0^+} \frac{-\sin^2 h \cdot h}{\frac{h^2}{1}} = 0$$

$$f'_-(0) = \lim_{h \rightarrow 0^-} \frac{f(0+h) - f(0)}{h} = \lim_{h \rightarrow 0^-} \frac{h+1-1}{h} = 1$$

$f'_+(0) \neq f'_-(0)$  olduğundan  $x=0$  da türevi mevcut

değildir.

$$35) \quad g(0) = 8 \quad , \quad f(x) = x \cdot g(x)$$

$$f'(0) = \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \rightarrow 0} \frac{h \cdot g(h) - 0 \cdot g(0)}{h}$$

$$f'(0) = \lim_{h \rightarrow 0} \frac{h \cdot g(h)}{h} = g(0) = 8 \quad //$$

$$36) \quad x \geq 0 \quad , \quad f(x) = \frac{\pi}{4} + \arctan \sqrt{e^{2x} - 1} - \arccos e^{-x}$$

$$f'(x) = \frac{\frac{2e^{2x}}{2\sqrt{e^{2x}-1}}}{1+e^{2x}-1} - \frac{e^{-x}}{\sqrt{1-e^{-2x}}} = \frac{1}{\sqrt{e^{2x}-1}} - \frac{e^{-x}}{e^x \sqrt{e^{2x}-1}} = 0$$

$f'(x) = 0 \Rightarrow f(x)$  sabit bir fonksiyondur.

$$x=0 \Rightarrow f(x) = \frac{\pi}{4} \quad //$$

$$37) \quad f\left(\frac{\pi}{2}\right) = 6 \quad , \quad f'\left(\frac{\pi}{2}\right) = 3 \quad , \quad g(x) = [f(x)]^{\sin x}$$

$$g(x) = [f(x)]^{\sin x}$$

$$g\left(\frac{\pi}{2}\right) = f\left(\frac{\pi}{2}\right)^{\sin \frac{\pi}{2}} = 6$$

$$\ln g(x) = \sin x \cdot \ln(f(x))$$

$$\frac{g'(x)}{g(x)} = \cos x \cdot \ln(f(x)) + \sin x \cdot \frac{f'(x)}{f(x)}$$

$$\frac{g'\left(\frac{\pi}{2}\right)}{g\left(\frac{\pi}{2}\right)} = \underbrace{\cos \frac{\pi}{2}}_0 \cdot \ln f\left(\frac{\pi}{2}\right) + \underbrace{\sin \frac{\pi}{2}}_1 \cdot \frac{f'\left(\frac{\pi}{2}\right)}{f\left(\frac{\pi}{2}\right)}$$

$$g'\left(\frac{\pi}{2}\right) = 6 \cdot \frac{3}{6} \Rightarrow g'\left(\frac{\pi}{2}\right) = 3 \quad //$$



$$38) \quad x^2 y^2 + \tan(x+y) - 1 = 0, \quad P\left(\frac{\pi}{4}, 0\right)$$

$$\frac{2xy^2}{0} + \frac{2x^2 y y'}{0} + (1+y')(1+\tan^2(x+y)) = 0$$

$$(1+y'_{1p}) \frac{(1+\tan^2 \frac{\pi}{4})}{2} = 0$$

$$1+y'_{1p} = 0 \Rightarrow \underline{y'_{1p} = -1}$$

$$\text{T.D.D. : } y - y_0 = f'(x_0)(x - x_0)$$

$$y - 0 = -1 \left(x - \frac{\pi}{4}\right)$$

$$y = -x + \frac{\pi}{4} //$$

$$39) \quad y \sin\left(\frac{1}{y}\right) + x \cos\left(\frac{1}{y}\right) = -2x, \quad P\left(0, \frac{1}{\pi}\right)$$

$$y' \cdot \frac{\sin\left(\frac{1}{y}\right)}{0} - y' \cdot \frac{1}{y} \cos\left(\frac{1}{y}\right) + \cos\left(\frac{1}{y}\right) + x \frac{y'}{y^2} \cdot \frac{\sin\left(\frac{1}{y}\right)}{0} + 2 = 0$$

$$y'_{1p} \pi - 1 + 2 = 0$$

$$y'_{1p} \pi = -1 \Rightarrow \underline{y'_{1p} = -\frac{1}{\pi}}$$

$$\text{T.D.D. : } y - \frac{1}{\pi} = -\frac{1}{\pi}(x - 0)$$

$$y = -\frac{1}{\pi}x + \frac{1}{\pi} //$$

$$40) \quad 2^{xy} + \ln\left[e + \arcsin \frac{y}{x}\right] = 1 + x, \quad P(1, 0)$$

$$\frac{(y + xy')}{0} \cdot 2^{xy} \cdot \ln 2 + \frac{\frac{y'x - y}{x^2}}{\sqrt{1 - \frac{y^2}{x^2}}} = 1$$

$$y'_{1p} \cdot \ln 2 + \frac{y'_{1p}}{e} = 1$$

$$y'_{1p} \left(\ln 2 + \frac{1}{e}\right) = 1$$

$$y'_{1p} = \frac{e}{e \ln 2 + 1}$$

$$\text{T.D.D. : } y - 0 = \frac{e}{e \ln 2 + 1} (x - 1)$$

$$y = \frac{e}{e \ln 2 + 1} x - \frac{e}{e \ln 2 + 1} //$$

$$41) \quad \cos(x-y) = x e^x \quad ; \quad P(0, \frac{\pi}{2})$$

$$\cos(x-y) - x e^x = 0$$

$$-(1-y') \cdot \sin(x-y) - e^x - \frac{x e^x}{0} = 0$$

$$(1-y'_{|P}) \cdot \sin \frac{\pi}{2} - 1 = 0$$

$$1 - y'_{|P} = 1$$

$$\underline{y'_{|P} = 0}$$

$$\text{T.D.D: } y - \frac{\pi}{2} = 0 \quad (x-0)$$

$$\underline{y = \frac{\pi}{2}} \quad //$$

$$42) \quad \sin(xy) = 1 - x^2 - y^2 + x^2 y^3$$

$$\sin 0 = 1 - x^2 - 0$$

$$1 - x^2 = 0 \Rightarrow \underline{x = \pm 1}$$

$$P_1(1, 0) \quad , \quad P_2(-1, 0)$$

$$(y + x y') \cos(xy) = -2x - \frac{2y y'}{0} + \frac{2x y^3}{0} + \frac{3x^2 y^2 y'}{0}$$

$$\underline{P_1(1, 0)}$$

$$m_1 = y'_{|P_1} = -2$$

$$\underline{P_2(-1, 0)}$$

$$m_2 = y'_{|P_2} = -2$$

$m_1 = m_2$  old. dan tegetler birbirine paraleldir.

$$43) \quad y - 2 \cos(\pi y - x) = 2x + 3 \quad , \quad P(0, 1)$$

$$y' + 2(\pi y' - 1) \sin(\pi y - x) = 2$$

$$y'_{|P} + 2(\pi \cdot y'_{|P} - 1) \sin(\pi - 0) = 2$$

$$y'_{|P} = 2$$

$$\text{T.D.D. : } y - 1 = 2(x - 0)$$

$$y = 2x + 1 \quad //$$

$$44) \sqrt[3]{28}$$

$$f(x) = \sqrt[3]{x}, \quad a = 27, \quad f(x) \approx L(x) = f(a) + f'(a)(x-a)$$

$$f'(x) = \frac{1}{3} x^{-2/3} \quad f(27) = 3, \quad f'(27) = \frac{1}{27}$$

$$\begin{aligned} \sqrt[3]{28} = f(28) &\approx L(28) = f(27) + f'(27)(28-27) \\ &= 3 + \frac{1}{27} = 3,037 \end{aligned}$$

$$\sqrt[3]{28} \approx 3,037 //$$

$$45) g(2) = -4, \quad g'(x) = \sqrt{x^2+5}, \quad g(2.05)$$

$$g(x) \approx L(x) = g(a) + g'(a)(x-a) \quad a = 2, \quad g'(2) = \sqrt{9} = 3$$

$$\begin{aligned} g(2.05) &\approx L(2.05) = g(2) + g'(2)(2.05-2) \\ &= -4 + 3(0,05) \\ &= -4 + 0,15 = -3,85 \end{aligned}$$

$$g(2.05) \approx -3,85 //$$

$$46) \sqrt[3]{(63)^2}$$

$$f(x) = \sqrt[3]{x^2}, \quad a = 64$$

$$f'(x) = \frac{2}{3} x^{-1/3} \quad f(64) = 16, \quad f'(64) = \frac{1}{6}$$

$$\begin{aligned} f(63) &\approx L(63) = f(64) + f'(64)(63-64) \\ &= 16 + \frac{1}{6}(-1) = 16 - 0,16 = 15,84 \end{aligned}$$

$$\sqrt[3]{(63)^2} \approx 15,84 //$$

$$47) (1,001)^5 - 3(1,001)^{7/3} + 2$$

$$f(x) = x^5 - 3x^{7/3} + 2, \quad a=1,$$

$$f'(x) = 5x^4 - 7x^{4/3}, \quad f(1) = 0, \quad f'(1) = -2$$

$$dy \approx \Delta y \Rightarrow f'(a) dx \approx f(a + \Delta x) - f(a)$$

$$dx = \Delta x = 1,001 - 1 = 0,001$$

$$f'(1) \cdot (0,001) \approx f(1,001) - f(1)$$

$$f(1,001) \approx -0,002 //$$

$$48) g(1) = g'(1) = 4 \quad f(x) = \frac{g(x^2)}{1+x^2}, \quad f(1,25)$$

$$a=1, \quad f(1) = \frac{g(1)}{2} = 2, \quad f'(x) = \frac{2x g'(x^2)(1+x^2) - 2x g(x^2)}{(1+x^2)^2}$$

$$f'(1) = \frac{4g'(1) - 2g(1)}{4}$$

$$f'(1) = \frac{16 - 8}{4} = 2$$

$$f(1,25) \approx L(1,25) = f(1) + f'(1)(1,25 - 1)$$

$$= 2 + 2(0,25) = 2,5$$

$$f(1,25) \approx 2,5 //$$

$$49) f(x) = (x^3 + x - 1)^7 + \arctan(x^4 - 1), \quad x=1, \quad f(1,02)$$

$$f'(x) = 7(x^3 + x - 1)^6(3x^2 + 1) + \frac{4x^3}{1+(x^4-1)^2}$$

$$f(1) = 1, \quad f'(1) = 28 + 4 = 32$$

$$f(1,02) \approx L(1,02) = f(1) + f'(1)(1,02 - 1)$$

$$= 1 + 32 \cdot 0,02 = 1,64$$

$$f(1,02) \approx 1,64 //$$

$$50) \quad g(1) = h'(1) = g'(1) = h(1) = 2, \quad f(x) = [g(x^2)]^{h(x)}$$

$$f(x) = [g(x^2)]^{h(x)}, \quad f(1) = (g(1))^{h(1)} = 2^2 = 4$$

$$\ln f(x) = h(x) \cdot \ln g(x^2)$$

$$\frac{f'(x)}{f(x)} = h'(x) \cdot \ln g(x^2) + h(x) \cdot \frac{2x g'(x^2)}{g(x^2)}$$

$$\frac{f'(1)}{f(1)} = h'(1) \cdot \ln g(1) + h(1) \cdot \frac{2g'(1)}{g(1)}$$

$$f'(1) = 4 \cdot \left( 2 \cdot \ln 2 + 2 \cdot \frac{2 \cdot 2}{2} \right)$$

$$f'(1) = 8 \ln 2 + 16 \quad //$$

$$51) \quad f(x) = \frac{1+x}{\sqrt{1+x^2}}, \quad f: (-\infty, 1] \rightarrow \mathbb{R}$$

$$f'(x) = \frac{\sqrt{1+x^2} - (1+x) \cdot \frac{1}{2} \cdot 2x (1+x^2)^{-1/2}}{1+x^2}$$

$$f'(x) = \frac{1+x^2 - x \cdot (1+x)}{(1+x^2)^{3/2}} = \frac{1+x^2 - x - x^2}{(1+x^2)^{3/2}} = \frac{1-x}{(1+x^2)^{3/2}} > 0$$

$f'(x) > 0 \Rightarrow f$ , 1-1 dir. Dolayısıyla tersi mevcuttur.

$$(f^{-1})'(0) = \frac{1}{f'(f^{-1}(0))}$$

$$\frac{1+x}{\sqrt{1+x^2}} = 0 \Rightarrow x = -1$$

$$f'(-1) = \frac{2}{2^{3/2}} = \frac{1}{\sqrt{2}}$$

$$(f^{-1})'(0) = \frac{1}{f'(-1)} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2} \quad //$$

$$52) f(x) = e^{\arctan x}$$

$$f'(x) = \frac{1}{1+x^2} e^{\arctan x} > 0 \quad e^{\arctan x} = e^{\pi/3}$$

$$\arctan x = \frac{\pi}{3} \Rightarrow x = \sqrt{3}$$

$f, 1-1$  dir ve tersi mevcuttur.

$$(f^{-1})'(e^{\pi/3}) = \frac{1}{f'(f^{-1}(e^{\pi/3}))} = \frac{1}{f'(\sqrt{3})}$$

$$= \frac{1}{1+3} e^{\arctan \sqrt{3}} = \frac{1}{4} e^{\pi/3} //$$

$$53) f(x) = 1+x + \ln(1+x^2)$$

$$f'(x) = 1 + \frac{2x}{1+x^2} = \frac{1+x^2+2x}{1+x^2} = \frac{(1+x)^2}{1+x^2} > 0 \Rightarrow 1-1 \Rightarrow \text{tersi var.}$$

$$(f^{-1})'(1) = \frac{1}{f'(f^{-1}(1))}$$

$$\begin{aligned} 1+x + \ln(1+x^2) &= 1 \\ x + \ln(1+x^2) &= 0 \end{aligned}$$

$$\Rightarrow x=0$$

$$f'(0) = \frac{1}{1} = 1$$

$$(f^{-1})'(1) = \frac{1}{f'(0)} = \frac{1}{1} = 1 //$$

$$54) y = f(x), x=1, 2x+y-1=0$$

$$2x+y-1=0 \Rightarrow y = -2x+1 \Rightarrow m_N = -2$$

$$m_N \cdot m_T = -1 \Rightarrow \boxed{m_T = \frac{1}{2} = f'(1)}$$

$$x=1 \Rightarrow 2 \cdot 1 + y - 1 = 0$$

$$y = -1$$

$$f(1) = -1$$

$$(f^{-1})'(-1) = \frac{1}{f'(1)} = \frac{1}{\frac{1}{2}} = 2 //$$

$$55) \quad f(x) = 2 + \arctan x + e^{2x}$$

$$f'(x) = \frac{1}{1+x^2} + 2e^{2x} > 0 \Rightarrow 1-1 \Rightarrow \text{tersi memcut}$$

$$2 + \arctan x + e^{2x} = 3 \Rightarrow \underline{x=0}, \quad f'(0) = 1+2=3$$

$$(f^{-1})'(3) = \frac{1}{f'(f^{-1}(3))} = \frac{1}{f'(0)} = \frac{1}{3} //$$

$$56) \quad g(t) = t^3 + 7t + 21, \quad g(-2) = -1$$

$$g'(t) = 3t^2 + 7, \quad g^{-1}(-1) = -2$$

$$g^{-1}(t) \approx L(t) = g^{-1}(-1) + (g^{-1})'(-1)(x+1)$$

$$(g^{-1})'(-1) = \frac{1}{g'(g^{-1}(-1))} = \frac{1}{g'(-2)} = \frac{1}{12+7} = \frac{1}{19}$$

$$g^{-1}(t) \approx L(t) = -2 + \frac{1}{19}(x+1) //$$

$$57) \quad f(x) = (\cos x^4)^{\arctan x^2}, \quad f'(0)$$

$$\ln f(x) = \arctan x^2 \ln(\cos x^4), \quad f(0) = 1$$

$$\frac{f'(x)}{f(x)} = \frac{2x}{1+x^4} \cdot \ln(\cos x^4) + \arctan x^2 \cdot \frac{-4x^3 \cdot \sin x^4}{\cos x^4}$$

$$\frac{f'(0)}{f(0)} = 0 \Rightarrow f'(0) = 0 //$$