Lecture 8

DC Circuits

Adapted from Prof. C.K.Tse

Fundamental Quantities

- Voltage — potential difference bet. 2 points
  - "across" quantity
  - analogous to 'pressure' between two points
- Current — flow of charge through a material
  - "through" quantity
  - analogous to fluid flowing along a pipe

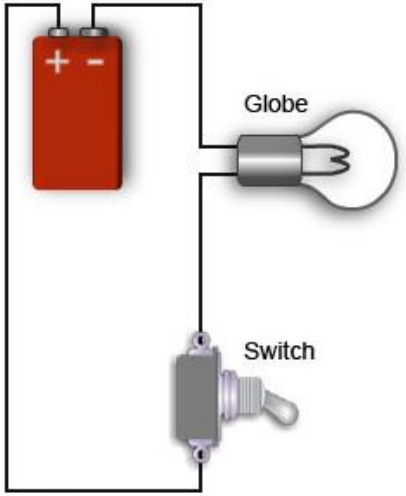


Figure 1: Electrical

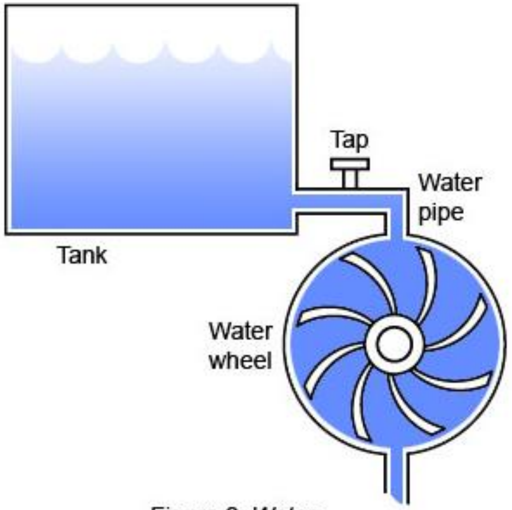


Figure 2: Water

Units of Measurement

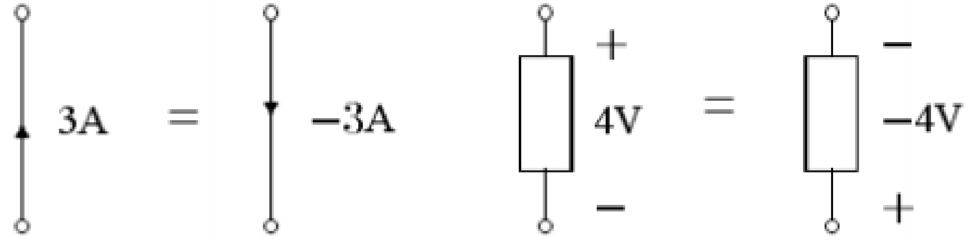
Voltage: *Volt* (V)
Current: *Ampere* (A)

Prefix	Multiplier (Abbreviation)
Peta	x 10^{15} (P)
Tera	x 10^{12} (T)
Giga	x 10^{9} (G)
Mega	x 10^{6} (M)
Kilo	x 10^{3} (K)
Hecto	x 10^{2} (h)
Deca	x 10 (da)
deci	x 10^{-1} (d)
centi	x 10^{-2} (c)
mili	x 10^{-3} (m)
micro	x 10^{-6} (μ)
nano	x 10^{-9} (n)
pico	x 10^{-12} (p)
femto	x 10^{-15} (f)

Direction and Polarity

- Current direction indicates the direction of flow of positive charge
- Voltage polarity indicates the relative potential between 2 points: + assigned to a higher potential point; and – assigned to a lower potential point.

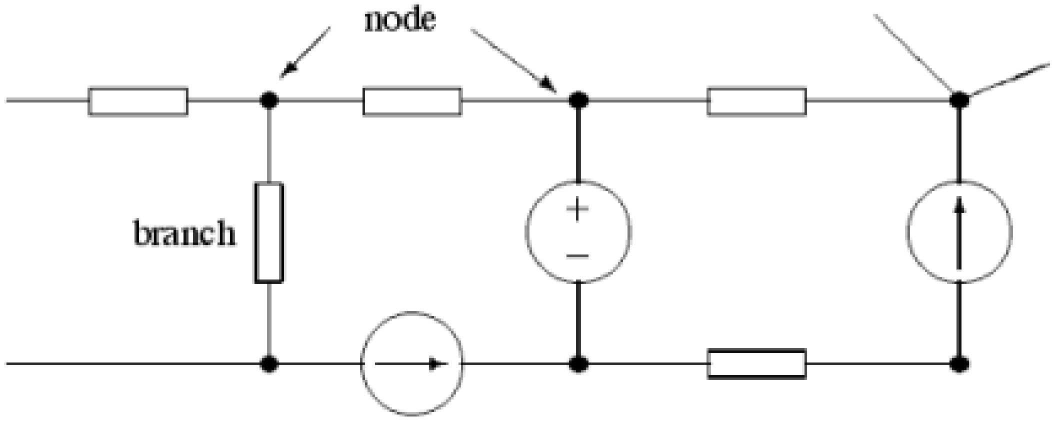
NOTE: Direction and polarity are arbitrarily assigned on circuit diagrams. Actual direction and polarity will be governed by the sign of the value.



Circuit

Collection of devices such as sources and resistors in which terminals are connected together by conducting wires.

- These wires converge in *NODES*
- The devices are called *BRANCHES* of the circuit

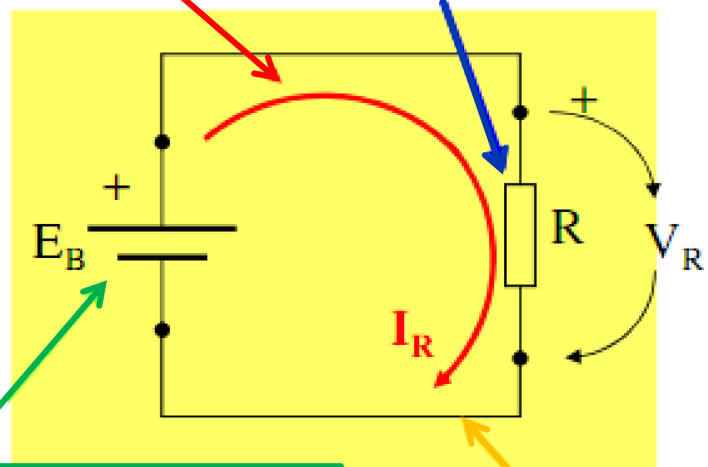


Circuit Analysis Problem:
Finding all currents and voltages in the branches of the circuit when the intensities of the sources are known.

Simplest DC Circuit

Current flows through the loop from higher voltage to lower voltage.

Symbol of resistor with its value (R) in ohm. Resistor current I_R flows through it, drops voltage, dissipates power and gets heated. It is a *load* to the battery

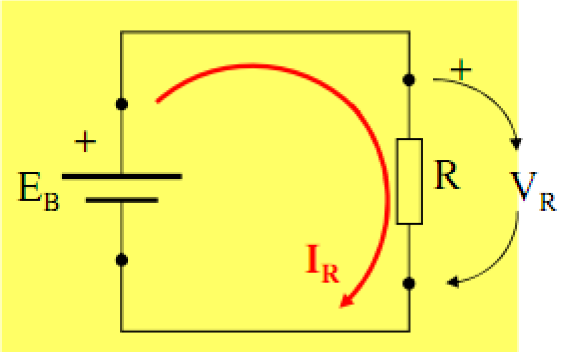


Voltage drop across resistor from higher to lower voltage.

Symbol of battery with polarity and value (E_B) in volt. Battery is a voltage source which ideally supplies *constant voltage* for all working condition.

Connecting wire; ideally, has no resistance and drops no voltage

Ohm's Law and Power Equation



Current through R, I_R *Ampere* (A)

Voltage drop across R, $V_R = I_R.R$ *Volts* (V) …….(Ohms Law)

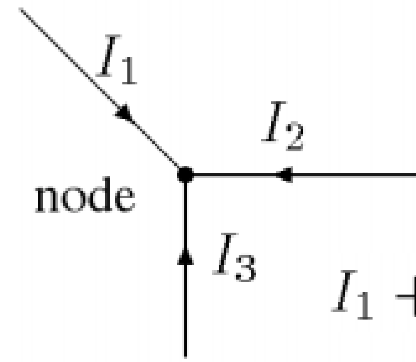
Power Dissipation in R, $P_R = V_R.I_R = I_R^2.R$ *Watt* (W)

Energy dissipated in time T sec, $W_R = P_R.T$ *Joules* (J)

(Another unit of energy is kW-hour (*kWhr*), 1 *kWhr* = 3,600,000 *Joules*)

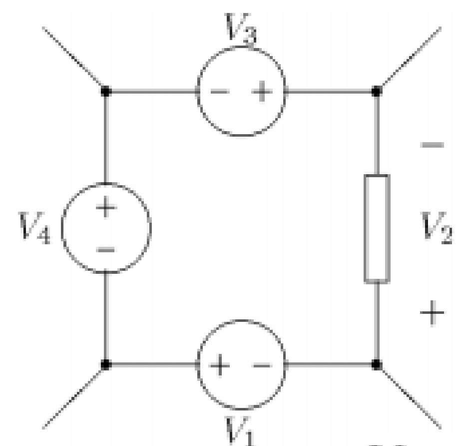
Kirchhoff's Laws

<u>*Kirchhoff's current law (KCL)*</u>
The algebraic sum of the currents in all branches which converge to a common node is equal to zero.



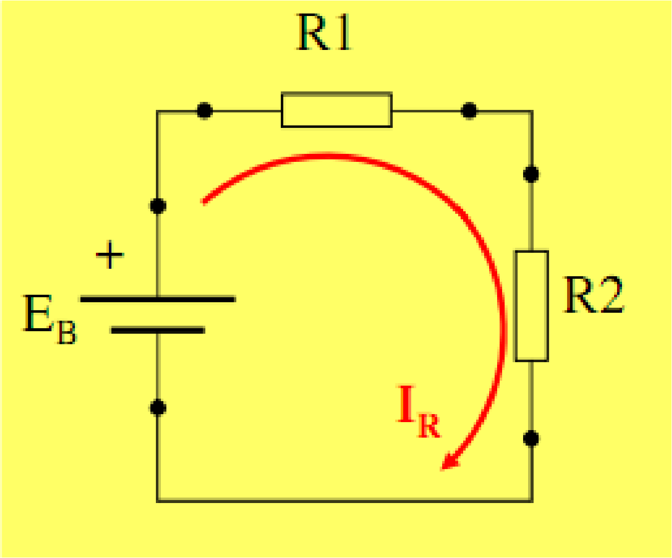
$$I_1 + I_2 + I_3 = 0$$

<u>*Kirchhoff's voltage law (KVL)*</u>
The algebraic sum of all voltages between successive nodes in a closed path in the circuit is equal to zero.



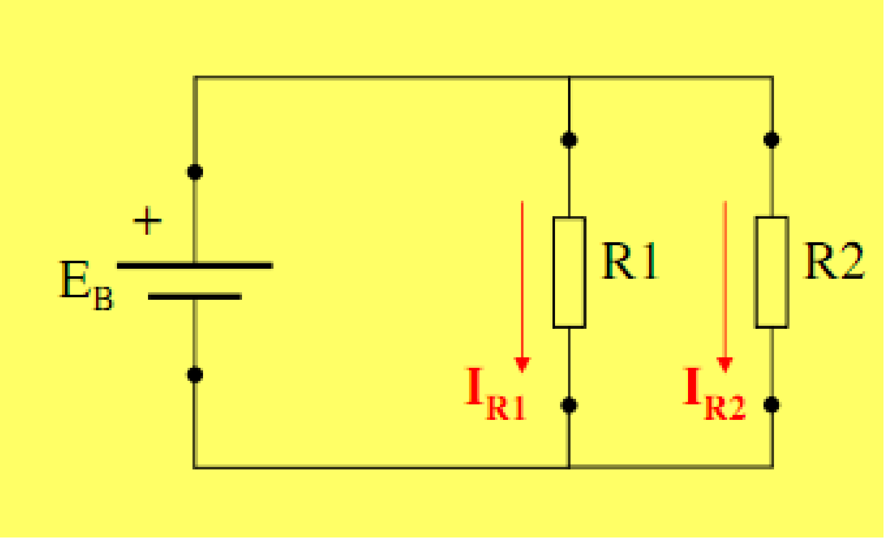
$$V_1 + V_2 + V_3 + V_4 = 0$$

Series Resistor Circuit



- Series circuit connects components in a single chain and current has only one path to flow.
- Series circuit makes a voltage divider circuit - R1 and R2 divides the battery voltage between themselves.

Parallel Resistor Circuit

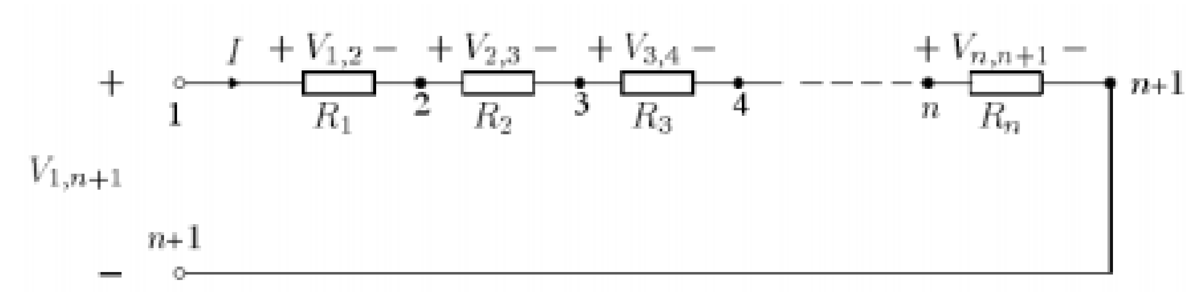


- Parallel circuit connects components in a way that current path is divided into two (or more) and then merged together again. This is a current divider circuit. R1 and R2 divides total battery current between themselves.

Series/Parallel Reduction

Series circuit

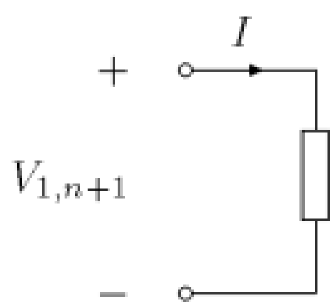
Each node is incident to just two branches of the circuit



KVL gives

$$\begin{aligned} V_{1,n+1} &= V_{12} + V_{23} + \cdots + V_{n,n+1} \\ &= (R_1 + R_2 + \cdots + R_n) I \end{aligned}$$

Hence, the equivalent resistance is:

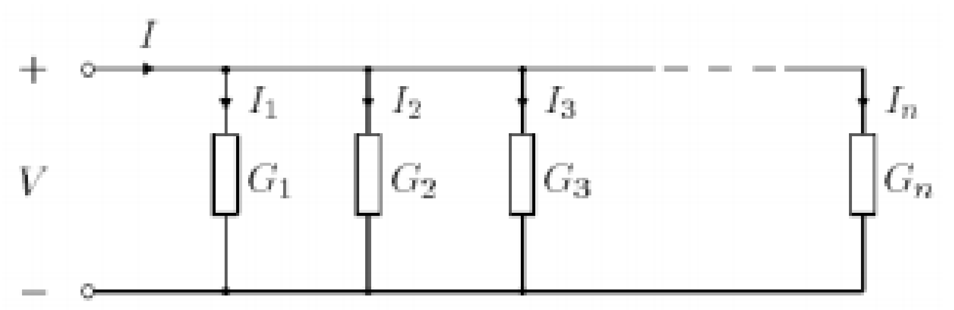


$$R = R_1 + R_2 + \cdots + R_n$$

Series/Parallel Reduction (cont.)

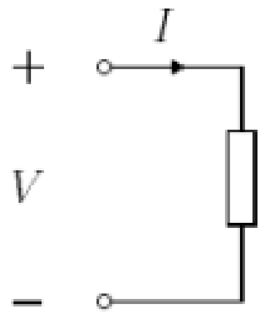
Parallel circuit

One terminal of each element is connected to a node of the circuit while other terminals of the elements are connected to another node of the circuit



KCL gives $I = (G_1 + G_2 + \cdots + G_n)V$

Hence, the equivalent resistance is:



$$G = G_1 + G_2 + \cdots + G_n$$

Note on Algebra

For algebraic brevity and simplicity:

- For series circuits, R is preferably used.
- For parallel circuits, G is preferably used.

For example, if we use R for the parallel circuit, we get the equivalent resistance as

$$R = \frac{R_1 R_2 R_3 \cdots R_n}{R_2 R_3 \cdots R_n + R_1 R_3 R_4 \cdots R_n + \cdots + R_1 R_2 \cdots R_{n-1}}$$

which is more complex than the formula in terms of G:

$G = G_1 + G_2 + \ldots + G_n$

Voltage/Current Division

For the series circuit, we can find the voltage across each resistor by the formula:

$$V_{i,i+1} = R_i I = \frac{R_i V}{R_1 + R_2 + \cdots + R_n}$$

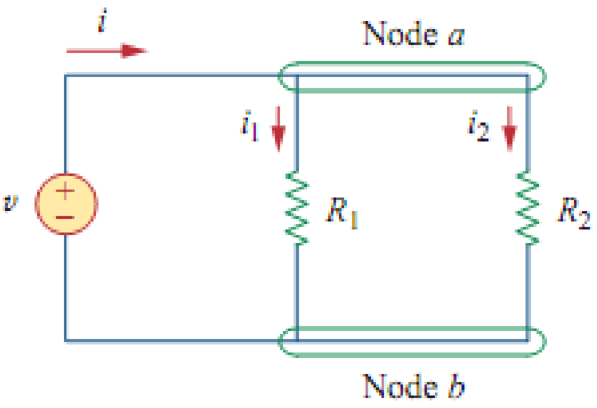
For the parallel circuit, we can find the current through each resistor by the formula:

$$I_i = G_i V = \frac{G_i I}{G_1 + G_2 + \cdots + G_n}$$

Note the choice of R and G in the formula!

Example

Find the currents i_1 and i_2 in terms of resistors and current i.

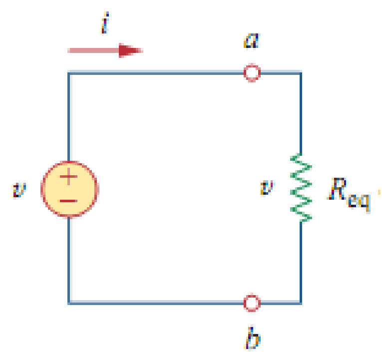


Solution:

$$v = i_1 R_1 = i_2 R_2$$

$$i_1 = \frac{v}{R_1}, \qquad i_2 = \frac{v}{R_2}$$

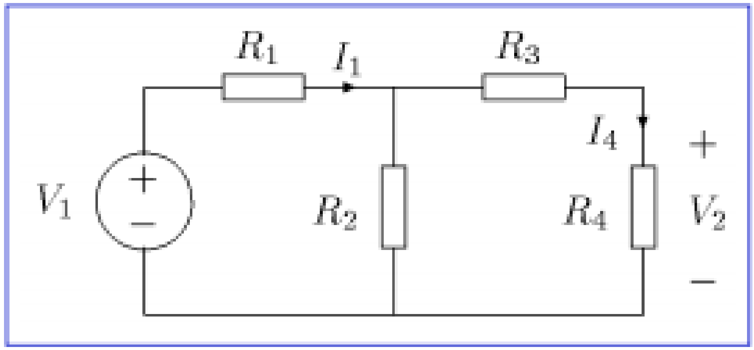
$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$



If $v = iR_{eq} = \frac{iR_1 R_2}{R_1 + R_2}$ *and* $i_1 = \frac{v}{R_1}$, $i_2 = \frac{v}{R_2}$ *then* $i_1 = \frac{R_2\, i}{R_1 + R_2}$, $i_2 = \frac{R_1\, i}{R_1 + R_2}$

Example

Consider this circuit, which is created deliberately so that you can solve it using series/parallel reduction technique. Find V_2.



Solution:

Resistance seen by the voltage source is

$$R = \frac{V_1}{I_1} = R_1 + \frac{1}{\frac{1}{R_2} + \frac{1}{R_3 + R_4}} = R_1 + \frac{R_2(R_3 + R_4)}{R_2 + R_3 + R_4}$$

Hence,

$$I_1 = \frac{(R_2 + R_3 + R_4)V_1}{(R_3 + R_4)(R_1 + R_2) + R_1 R_2}$$

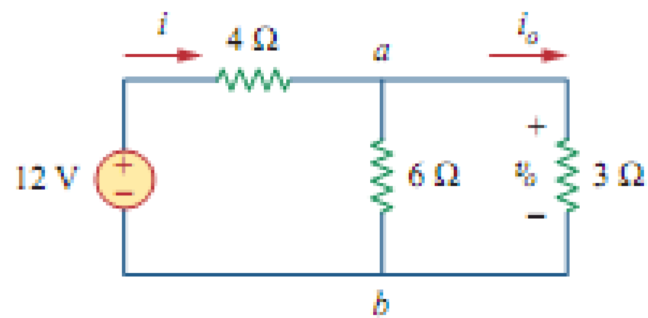
Current division gives:

$$I_4 = I_1 \times \frac{\left(\frac{1}{R_3 + R_4}\right)}{\left(\frac{1}{R_3 + R_4}\right) + \frac{1}{R_2}} = \frac{R_2 I_1}{R_2 + R_3 + R_4}$$

Then, using $V_2 = I_4 R_4$, we get

$$V_2 = \frac{R_2 R_4 V_1}{(R_1 + R_2)(R_3 + R_4) + R_1 R_2}$$

Example

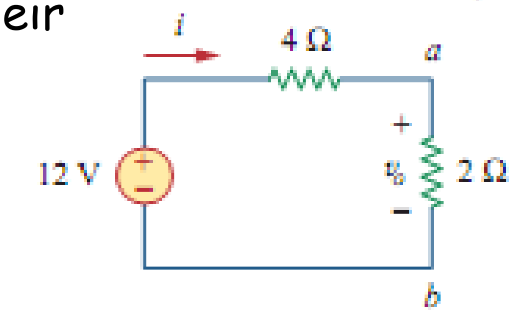


Find i_0 and v_0 in the circuit shown in figure. Calculate the power dissipated in the 3Ω resistor.

<u>*Solution:*</u>

The 6 Ω and 3 Ω resistors are in parallel, so their combined resistance is

$$6\ \Omega \parallel 3\ \Omega = \frac{6 \times 3}{6 + 3} = 2\ \Omega$$



We can obtain v_0 by using Ohm's law

$$i = \frac{12}{4 + 2} = 2\ \text{A}$$

By applying current division to the equivalent circuit, now that we know i, by writing

$$i_o = \frac{6}{6 + 3} i = \frac{2}{3}(2\ \text{A}) = \frac{4}{3}\ \text{A}$$

The power dissipated in the 3 Ω resistor is

$$p_o = v_o i_o = 4\left(\frac{4}{3}\right) = 5.333\ \text{W}$$