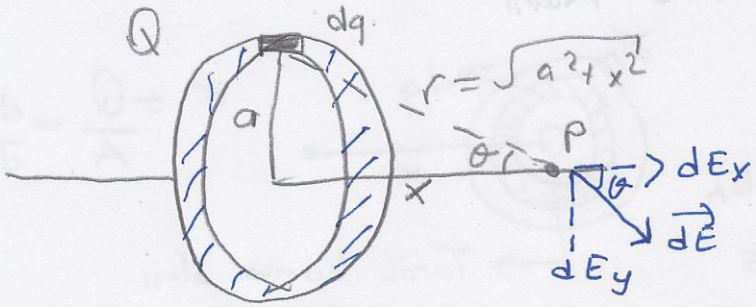
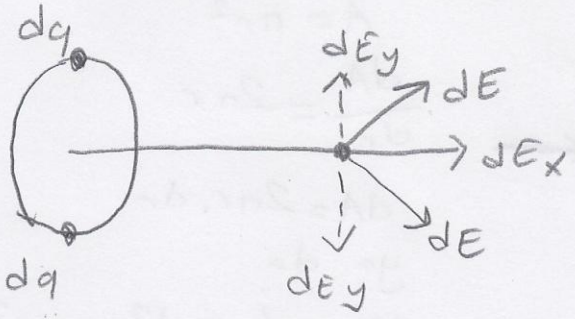
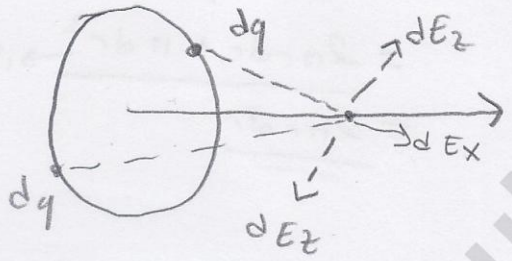
Düzgün Yüklü Bir Halkanın Elektrik Alanı



$$dE = k\cdot\frac{dq}{r^2}$$



Simetriden $\int d\vec{E_y} = 0$



$\int d\vec{E_z} = 0$

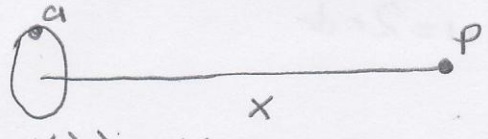
$$E_p = \int dE_x = \int dE\cdot\cos\theta = \int\left(k\frac{dq}{r^2}\right)\cdot\left(\frac{x}{r}\right) = \int k\cdot\frac{dq\cdot x}{r^3}$$

$r^3 \rightarrow (a^2+x^2)^{3/2}$

$$= k\cdot\frac{x}{(a^2+x^2)^{3/2}}\int dq$$

$\int dq$: toplam yük, Q

$$\boxed{\vec{E_p} = k\cdot\frac{Q\cdot x}{(a^2+x^2)^{3/2}}\cdot\vec{i}}$$

* 

(P çok uzakta)

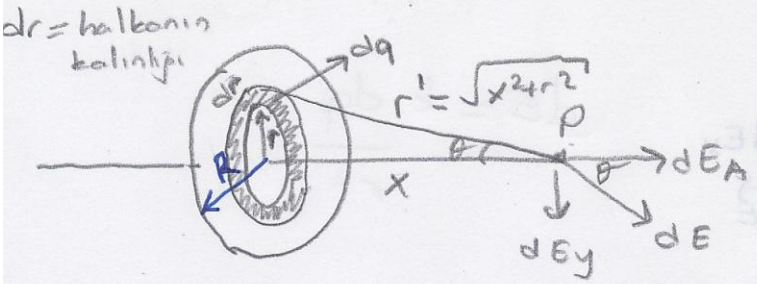
$x >> a$ ise

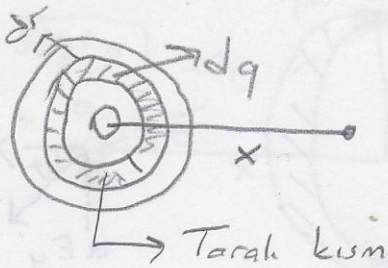
$$E_p = \frac{k\cdot Q\cdot x}{(a^2+x^2)^{3/2}} \approx k\cdot\frac{Q}{x^2}$$

nokta yük gibi

[redacted] =

Düzgün Yüklü Bir Diskin Elektrik Alanı





$$\sigma = \frac{Q}{A} = \frac{dq}{dA}$$

$$dE = k \cdot \frac{dq}{r'^2}$$

$$\sigma = \frac{Q}{A} = \frac{dq}{dA}$$

$$\sigma = \frac{Q}{(\pi R^2)} = \frac{dq}{2\pi r dr}$$

$$dq = \sigma(2\pi r dr)$$

$$A = \pi r^2$$

$$\frac{dA}{dr} = 2\pi r$$

$$dA = 2\pi r \cdot dr$$

ya da

$$dA = \pi(r+dr)^2 - \pi r^2$$

$$= 2\pi r dr + \pi dr^2 \rightarrow \text{ihmal}$$

$$= 2\pi r dr$$

$$E_p = \int dE_x = \int dE\cos\theta = \int \left(k \cdot \frac{dq}{r'^2}\right)\left(\frac{x}{r'}\right)$$

$$= \int k \cdot \frac{dq}{r'^3} \cdot x = \int k \frac{[\sigma(2\pi r dr)] \cdot x}{(x^2+r^2)^{3/2}}$$

$$= k\sigma\pi x \int \frac{2r\,dr}{(x^2+r^2)^{3/2}}$$

$u = x^2 + r^2$ dönüşümü

$$\frac{du}{dr} = 2r$$

$$du = 2r\,dr$$

$$= k\sigma\pi x \int \frac{du}{u^{3/2}}$$

$$= k\sigma\pi x \int u^{\frac{-3}{2}} du = k\sigma\pi x \left[\frac{u^{\frac{-3}{2}+1}}{\frac{-3}{2}+1}\right] = k\sigma\pi x \left[\frac{-2}{u^{1/2}}\right]$$

$$= 2k\sigma\pi x \left[\frac{-1}{(x^2+r^2)^{1/2}}\right]_0^R = 2k\sigma\pi x \left\{\left[\frac{-1}{(x^2+R^2)^{1/2}}\right] - \left[\frac{-1}{(x^2+0^2)^{1/2}}\right]\right\}$$

$$= 2k\sigma\pi x \left\{ \frac{1}{|x|} - \frac{1}{(x^2+R^2)^{1/2}} \right\}$$

$$\vec{E_p} = 2k\sigma\pi \left\{ \frac{x}{|x|} - \frac{x}{(x^2+R^2)^{1/2}} \right\} \cdot \vec{i}$$

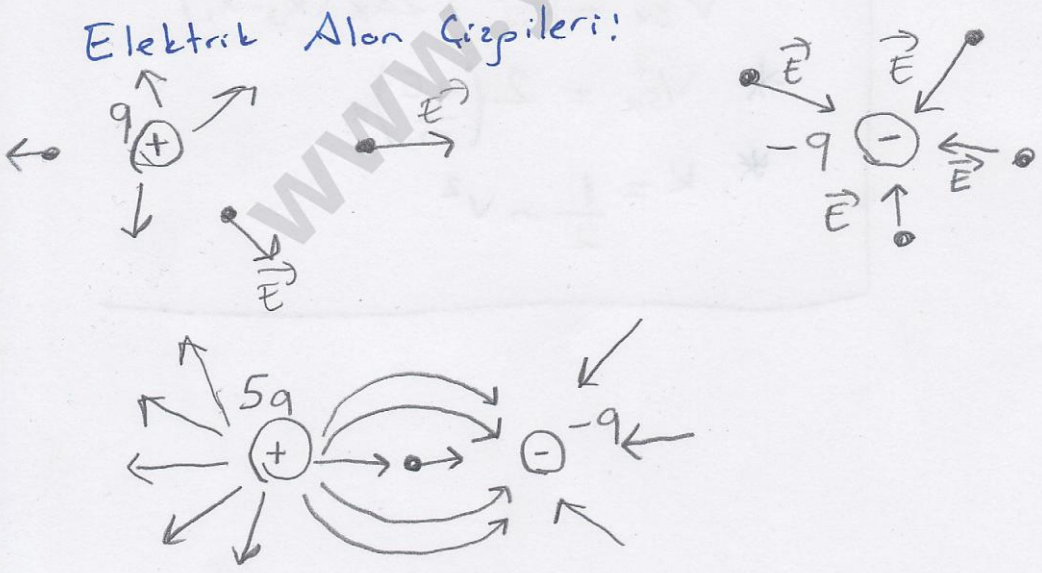
* $R >> x$ ise, yani diske çok yakın bir nokta için;

$$E_p = 2k\sigma\pi - \left\{ 1 - \frac{x}{(x^2+R^2)^{1/2}} \right\}$$

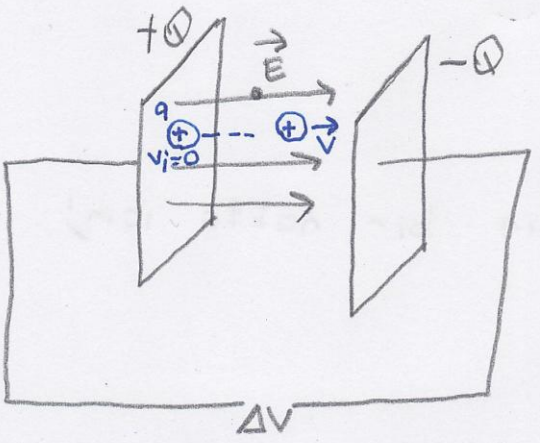
$$= 2\left(\frac{1}{4\pi\varepsilon_0}\right)\sigma\pi$$

$$E_p = \frac{\sigma}{2\varepsilon_0}$$ } yalıtkan bir disk için çok yakınında elektrik alanı

Elektrik Alan Çizgileri:



Düzgün Bir Elektrik Alanında Yüklü Bir Parçacığın Hareketi



$$\vec{F} = q.\vec{E} = m.\vec{a}$$

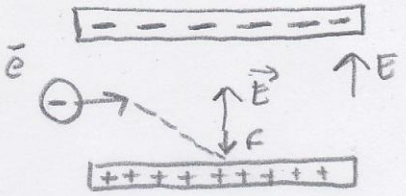
$$\vec{a} = q.\frac{\vec{E}}{m}$$

$$x_s = x_i + V_{xi}t + \frac{1}{2}a_x t^2$$

$x_i \to 0$, $V_{xi} \to 0$

$$x = \frac{1}{2}at^2$$

$$* \quad x = \frac{1}{2}\left[\frac{q.E}{m}\right]t^2$$



$$\vec{F} = -e\vec{E} = m.\vec{a}$$

$$\vec{a} = -\left(\frac{eE}{m}\right)\vec{j}$$

$-y$

$$V_{sx} = V_{ix} + a_x t$$

$$* \quad V_s = at = \left(\frac{qE}{m}\right)t$$

$$V_{sx}^2 = V_{ix}^2 + 2a_x(x_s - x_i)$$

$$* \quad V_{sx}^2 = 2\left(\frac{qE}{m}\right)x$$

$$* \quad K = \frac{1}{2}mv^2$$

$$* \quad V_{ix} = V_i = \text{sabit}$$

$$V_{iy} = 0$$

$$* \quad V_{sy} = V_{iy} + a_y t$$

$$* \quad V_{sy} = -\left(\frac{eE}{m}\right)t\,\vec{j}$$

$$x = x_i + V_{xi}t + \frac{1}{2}a_x t^2$$

$$x = V_{xi}t = v_i t$$

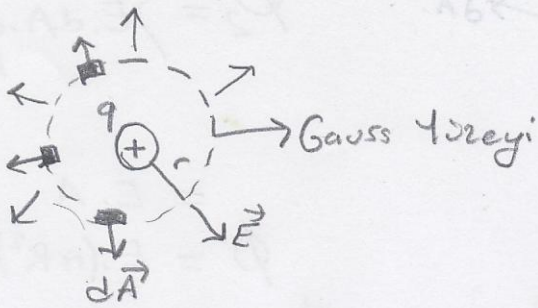
$$* \quad y_s = \underbrace{y_i}_{0} + \underbrace{V_{yi}t}_{0} + \frac{1}{2}a_y t^2 = \frac{1}{2}a_y t^2 = \frac{-1}{2}\left(\frac{eE}{m}\right)t^2.\vec{j}$$

Gauss Yasası

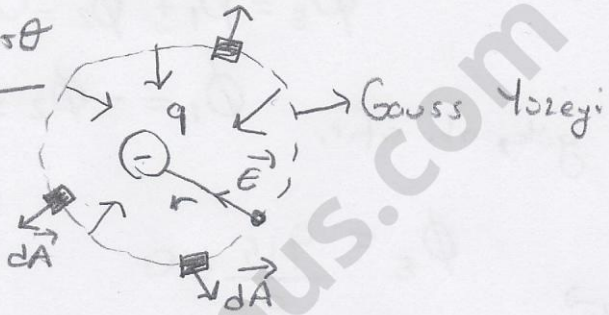
→ Elektrik Akısı:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{iç}}{\varepsilon_0}$$

kapalı yüzey integral



* $\Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E \cdot dA \cdot \cos\theta$



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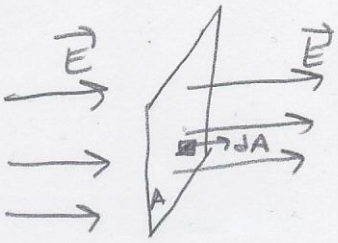
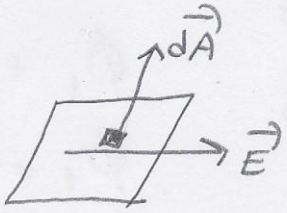


$$\Phi_E = \int E \cdot dA \cdot \underbrace{\cos\theta}_{1^\circ} = E \int dA$$

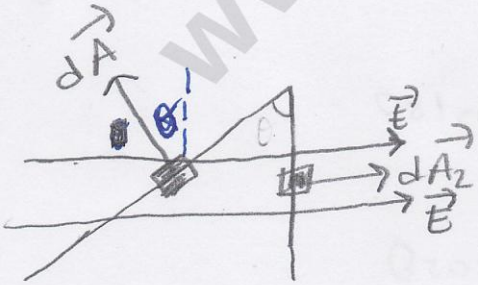
$$\Phi_E = E \cdot A$$

*



$$\Phi = \int E\,dA \cdot \underbrace{\cos 90}_{0} = 0$$

*



$$\Phi_1 = \int E \cdot dA \cdot \cos(90+\theta)$$

$$= E \int dA_1 \cdot \cos(90+\theta)$$

$$\rightarrow \Phi_1 = E \cdot A_1 \cdot \sin\theta$$

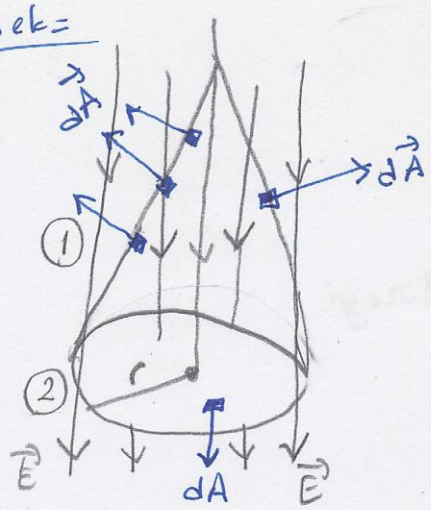
$$\Phi_2 = \int E \cdot dA \cdot \cos\theta$$

$$= E \int dA_2 \cdot \underbrace{\cos 0}_{1}$$

$$\rightarrow \Phi_2 = E \cdot A_2$$

Toplam Akı $= \Phi_E = \Phi_1 + \Phi_2$

Örnek=



$$\Phi_2 = \int E.dA.\cos\theta$$

($\cos\theta = 1$)

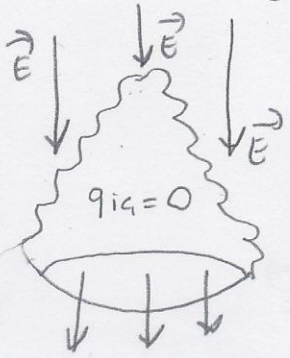
$$= E.A$$

$$\Phi = E.(\pi R^2)$$

$\Phi_E = \Phi_1 + \Phi_2 = 0$ = Bir yüzeyden girip diğerinden çıkıyo

$$\Phi_1 = -\Phi_2 = -E(\pi R^2)$$

* Koni içinde yük, akı sıfır.

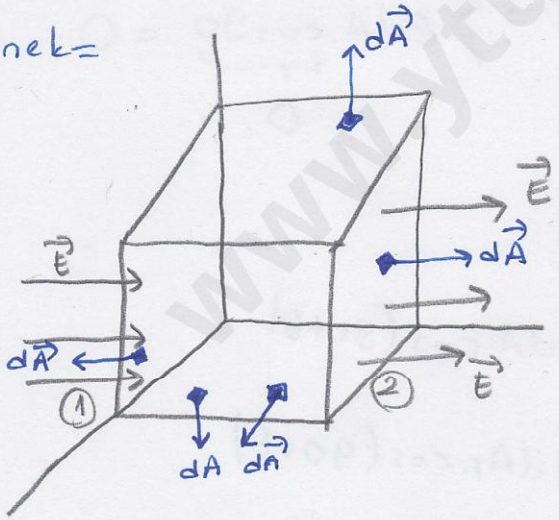


$$\Phi_E = \frac{q_{iç}}{\varepsilon} = 0$$

$$\Phi_1 + \Phi_2 = 0$$

$$\Phi_1 = -\Phi_2$$

Örnek=



Toplam elektrik akısı

$$\Phi_E = \Phi_1 + \Phi_2 + \Phi_3 + \Phi_4 + \Phi_5 + \Phi_6$$

$$= \int_{①} E\,dA.\cos 180 + \int_{②} E.dA.\cos 0 + \int_{③} E.dA.\cos 90 + \int_{④} E.dA.\cos 270 + \int_{⑤} E.dA.\cos 90 + \int_{⑥} E.dA.\cos 270$$

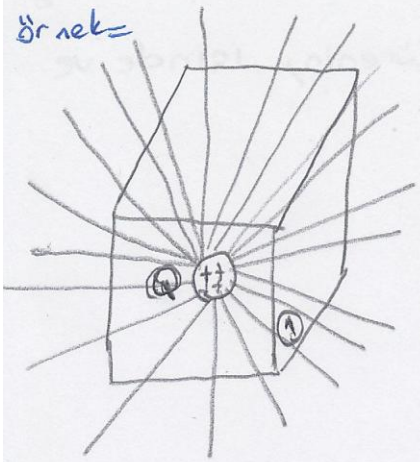
($\cos 90 = 0$, $\cos 270 = 0$)

$$\Phi_E = -E\int_{①} dA + E\int_{②} dA$$

$$= -E.A + E.A$$

$$\Phi_E = 0 \qquad Q_{iç} = 0$$

Örnek=



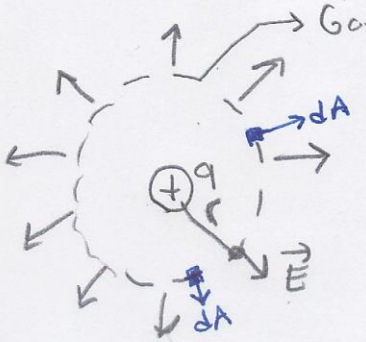
Kaban bir yüzeyi için elektrik akısı =?

Toplam akı $\Phi = \frac{q_{iç}}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$

Bir yüzey için;

$$\Phi_{\substack{1\\(2)\\(3)\\\vdots}} = \frac{\Phi_E}{6} = \frac{Q/\varepsilon_0}{6} = \frac{Q}{6\varepsilon_0}$$

Gauss Yasası



$$\Phi_E = \oint \vec{E}\cdot d\vec{A} = \oint E.dA.\underbrace{\cos 0}_{1}$$

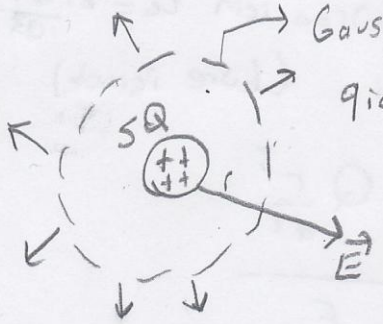
$$= E\oint dA = E.A$$

$$= \left(k\cdot\frac{q}{r^2}\right).(4\pi r^2)$$

$$= \left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}\right)(4\pi r^2)$$

$$Q_E = \frac{q}{\varepsilon_0} \;///$$

Örnek=



$q_{iç} = 5Q$

$E = ?$

$$Q_E = \oint \vec{E}.d\vec{A} = \frac{q_{iç}}{\varepsilon_0}$$

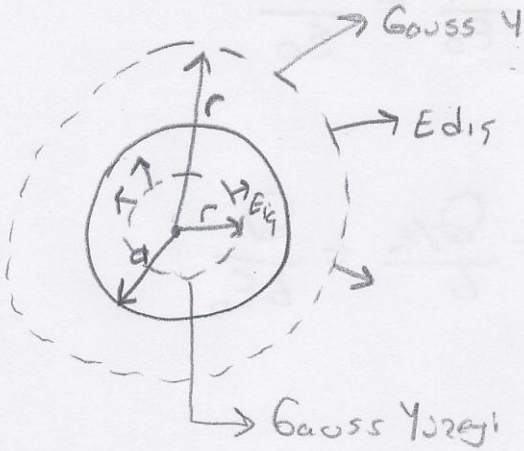
$$E\oint dA = \frac{5Q}{\varepsilon_0} \rightarrow$$

$$E.(4\pi r^2) = \frac{5Q}{\varepsilon_0}$$

$$\vec{E} = \left(\frac{5Q}{4\pi\varepsilon_0 r^2}\right)\hat{r} = \left(k\frac{5Q}{r^2}\right)\hat{r}$$

///

Örnek= Düzgün yük yoğunluklu, yalıtkan bir kürenin, içinde ve dışındaki bir noktada elektrik alanı=?



r > a için

$q_{iç} = Q$

r < a için

$q_{iç} = ?$

$q_{iç} = ?$

$\rho = \frac{Q}{V} = \frac{q_{iç}}{V_{iç}}$

$\frac{Q}{\frac{4}{3}\pi a^3} = \frac{q_{iç}}{\frac{4}{3}\pi r^3} \rightarrow q_{iç} = Q.\frac{r^3}{a^3}$

$\Phi_E = \oint E.dA.\cos 0 = \frac{q_{iç}}{\varepsilon_0}$

$E \oint dA = \frac{q_{iç}}{\varepsilon_0}$

a) r > a için (küre dışında)

$E(4\pi r^2) = \frac{Q}{\varepsilon_0}$

$E = \frac{Q}{4\pi\varepsilon_0 r^2}$

$\vec{E}_{dış} = \left(k.\frac{Q}{r^2}\right)\hat{r}$

b) Küre yüzeyinde r=a'da

$E_a = ?$

$E_{dış} = k.\frac{Q}{r^2}$ yada $\Rightarrow r=a$ için $E_a = k.\frac{Q}{a^2}$

$E_{iç} = \frac{k.Q.r}{a^3} \Rightarrow r=a$ için $E_a = k.\frac{Q.a}{a^3}$

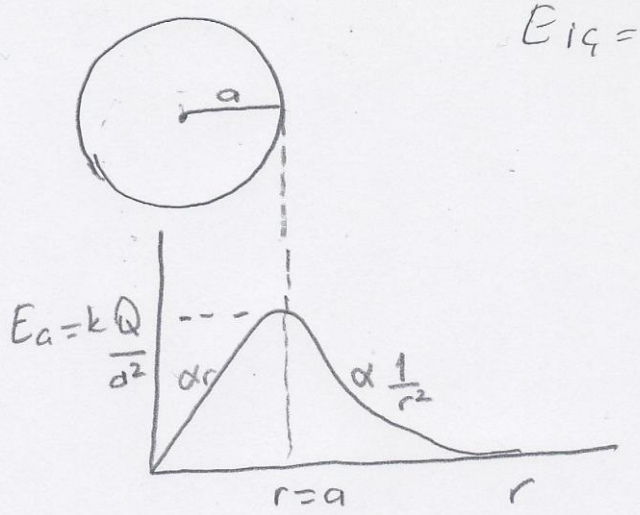
c) r < a için (küre içinde)

$E(4\pi r^2) = \frac{Q\frac{r^3}{a^3}}{\varepsilon_0}$

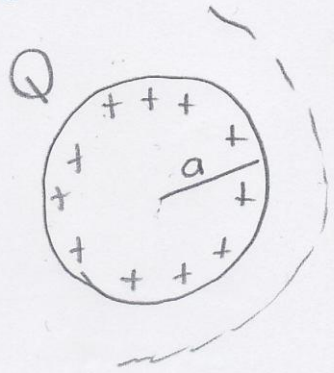
$\vec{E}_{iç} = \frac{Q.r}{4\pi.\varepsilon_0.a^3} = \left(k.\frac{Q}{a^3}.r\right)\hat{r}$

örnek=

$E_{iç} = \frac{kQ}{a^3} r$ $\quad E_{dış} = k \cdot \frac{\phi}{r^2}$



örnek=



İletken küre için;

$r < a$'da (küre içinde) $E_{iç} = 0$

$q_{iç} = 0$

$r > a$'da (küre dışında)

$E_{dış} = \frac{kQ}{r^2}$