Nümerik Analiz
2.Hafta Ders Notu

2) Regula falsi Method

→ Step 1 = Check $f(a).f(b)$. If $f(a).f(b) < 0$, this method can be implied.

→ Step 2 = Find
$$c_k = \frac{f(b_k).a_k - f(a_k).b_k}{f(b_k) - f(a_k)}$$
where
k: iteration number

→ Step 3 = If $f(c_k)$ is zero, c_k is a root
Else

→ Step 4 = If $f(a_k)$ and $f(c_k)$ have the same sign then
we set $a_{k+1} = c_k$, $b_{k+1} = b_k$
otherwise

→ Step 5 = We set $a_{k+1} = a_k$, $b_{k+1} = c_k$

Ex. find a root of $f(x) = x^2 - 3$, $[1,2]$ $\varepsilon = 0,01$

sol.

$$c_k = \frac{f(b_k) \cdot a_k - f(a_k) \cdot b_k}{f(b_k) - f(a_k)}$$

Sol	a	b	f(a)	f(b)	c	f(c)	uptade
#1	1	2	−2	1	5/3 = 1,66	−0,2221	a = c
#2	1,6667	2	−0,2221	1	1,7273	−0,00164	a = c
#3	1,7273	2	−0,0164	1	1,7317	0,0012 < ε	

root = 1,7317

Ex. find the root of $f(x) = x^3 + 3x - 5$, $[1,2]$, $\varepsilon = 0{,}001$

sol.

$f(a) \cdot f(b) < 0$ ✓

	a	b	f(a)	f(b)	c	f(c)	uptade
#1	1	2	−1	9	1,1	−0,369	a = c
#2	1,1	2	−0,369	9	1.135447	−0.129796	a = c
#3	1.135447	2	−0,12976	9	1,147379	−0,044868	a = c
#4	1,147379	2	−0,0448689	9	1.1519657	−0,015416	a = c
#5	1,1519657	2	−0,015416	9	1,153416	−0,005285	a = c
#6	1,153416	2	−0,005285	9	1,153913	−0,001811	a = c
#7	1.153913	2	−0,001811	9	1,154083	−0,000620 < ε	

root = 1,154083

3. Newton – Raphson Method (NR)

In numerical analysis, NR method is a method for finding succesively better approximations to the roots of real-valued function which has a continuous derriative which we can compute.

The NR method in one variable is implemented as follows:

– Given a function f, defined over the reals (x) and its deriative f'

– We begin with a first guess x_0 for a root of the function.

– Provided the function is reasonably well-behaved a better approximation x_1 is $x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$ 1. iteration

Geopetrically $(x_1, 0)$ is the intersection wit the x-axis of a line tangent of f at $(x_0, f(x_0))$

– The process is repeated as $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

$x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$ 2. iteration

$x_3 = x_2 - \dfrac{f(x_2)}{f'(x_2)}$ 3. iteration

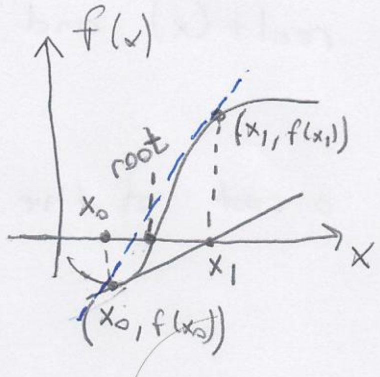
$\vdots$

Stopping criteria;

$|f(x_k)| < \varepsilon$ or $|x_{k+1} - x_k| < \varepsilon$

The NR method is much more efficient and faster than other methods. But the deriative of the function is very hard and also for this reason the guess value. (initial) is carefully selected.

→ for example $f(x) = \frac{20x-1}{19x}$, $x_0 = ?$



Ex

find the root of $f(x) = x^3 - 5$ by using NR method.

$(x_0 = 2 \quad , \quad \varepsilon = 0{,}001)$

Sol $f'(x) = 3x^2$

#1 $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{(2^3 - 5)}{3.(2^2)} = 1{,}75$

#2 $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1{,}75 - \frac{((1.75)^3 - 5)}{3.(1.75^2)} = 1.7108$

#3 $x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.709976$

#4 $x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 1{,}709976$

$x_4 - x_3 = 1{,}709976 - 1.709976 = 0 < \varepsilon$

$root = x_k = \underline{1{,}709976}$

Ex find the root $\cos x = x^3$ by using NR,

$x_0 = 0,5 \quad \mathcal{E} = 0,0001$

sol

(Radyan modunda)

$f(x) = \cos x - x^3 \quad f'(x) = -\sin x - 3x^2$

$$x_1 = 0,5 - \frac{\cos(0,5) - (0,5)^3}{-\sin(0,5) - 3(0,5)^2} = 1,1121416$$

$x_2 = 0,909672$

$x_3 = 0,867263$

$\left.\begin{array}{l} x_4 = 0,865477 \\ x_5 = 0,865474 \end{array}\right\} \checkmark \to x_5 - x_4 < \mathcal{E}$, root $= \underline{0,865477}$

Ex find the root $e^{x-3} = -x+2$ by using NR.

$(x_0 = 2, \ \mathcal{E} = 0,0001)$

sol $f(x) = e^{x-3} + x - 2 = 0, \quad f'(x) = e^{x+3} + 1$

$$x_1 = 2 - \frac{e^{2-3} + 2 - 2}{e^{2-3} + 1} = 1,731058$$

$$x_2 = 1,731058 - \frac{e^{1,731058} + 1,731058 - 2}{e^{1,731058}} = 1,72154$$

$\left.\begin{array}{l} x_3 = 1,72135 \\ x_4 = 1,72135 \end{array}\right\} x_4 - x_3 < \mathcal{E} \to$ root $= 1,72135$

4- Fixed-Point Method (Successive Substitution)

By rearranging the function $f(x)=0$ so that x is on the left-hand side of the equation;

$$x = g(x)$$

This transformation can be accomplished by algebric manipulation. The utility of equation is that it provides a formula to predict a new value of x as a function if an old value of x. Thus, given an initial guess at root x_i equation can be used to compute a new estimate x_{i+1} as expressed by the iterative formula

$$x_{i+1} = g(x_i)$$

$$x_{i+2} = g(x_i+1)$$

$$\vdots$$

Stopping creation $= |x_{i+1} - x_i| < \varepsilon$

When $0 \leq g'(x) < 1$ on (a,b), the convergence to the unique solution is quarented.

Ex find the root of $f(x) = x^3 - x - 3$ by using FM

$x_0 = 1,5 \qquad \varepsilon = 0,01$

Sol

1) $x = x^3 - 3 \xrightarrow{so} \rho(x) = x^3 - 3$

2) $x^3 = x + 3 \longrightarrow x = \sqrt[3]{x+3}$ so $\rho(x) = \sqrt[3]{x+3}$

$x = \dfrac{3}{x^2 - 1}$ so $\rho(x) = \dfrac{3}{x^2 - 1}$

	$x^3 - 3$	$\sqrt[3]{x+3}$	$\dfrac{3}{x^2-1}$
#1	0,375	1,651	2,4
#2	−2,947	1,669	0,63
#3	−28,5941	1,671	−4,974
#4	$-2,3382 \cdot 10^4$	1,672	0,126
#5	$-1,2784 \cdot 10^{-3}$	1,672	−3,046

$x_5 - x_4 < \varepsilon$, root 1,672
(x_4) //