1. Errors

The possible source of errors in a computation.

- Human Errors
- Truncation Error
- Rovading Error

These errors can be very hard to detect unless they give obviously incorrect solution. In this course, we shall assume that human errors are not present.

A truncation error is present when same in finite process is approximated by a finite process for example consider the Taylor series Expansion,

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!}$$

If $e^{0,1} \cong 1 + 0,1 + \frac{(0,1)^2}{2!} + \frac{(0,1)^3}{3!} \cong 1,105$

Rounding error =

- 3,265 or 3,27 or 3.3
- 14.12 or 14

Scientific error
- absolute error $|x_{real} - x_{computed}|$
- relative error $|x_{real} - x_{computed}|$

2. Solution of Equation of One Variable (single (Root finding)

The root of a function $f(x)$ $[f: R \to R]$ is simply, some value r for which the function is zero, that is, $f(r) = 0$

This topic is broken into two major sub-problems.

1. finding the root of a reel-valued function of a single variable

2. finding the root of a vector-valued function of a many variables.

This topic is concerned with finding scalar, real or complex root.

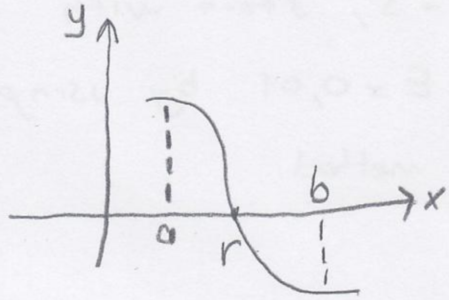
Numerical root-finding methods use steration, producing a sequence of numbers.

There are sone techniques which may be used to find the root of a univariate (one variable) function

1. Bisection Method
2. False-Position Method (Regula falsi)
3. Newton's method (Donivative based method)
4. Secant method
5. Müller's method
6. Combination of method (Brent's Method)

2.1 Bisection Method (Yarıya Bölme)

Problem = Given a function of one variable $f(x)$, find a value r (called root) such that $f(r)=0$



$r \in [a,b]$

Assuptions = We will assume that the function $f(x)$ is continuous.

Initial Requirements = We have aninitial band $[a,b]$ on the root that is $f(a)$ and $f(b)$ have opposite signs. ($f(a)$ and $f(r)$)

Process Steps

1) Given the interval $[a,b]$, determine ε (error tolerance)
2) Define $c = \frac{a+b}{2}$ ($\varepsilon = 0,01$ $\varepsilon = 0,001$ $\varepsilon = 0,0001$)
3) If $f(c)=0$
(Unlikely in practise) we have found a root.

4) If $f(a)$ and $f(c)$ have opposite signs, then a root must lie on $[a,c]$, so assign $b=c$

5) Else
If $f(a)$ and $f(c)$ have not opposite signs, then a root must lie on $[c,b]$, so assign $a=c$

b) Stopping criteria (k: iteration number)

1 - $f(c_k) < \varepsilon$

2 - $\dfrac{|C_{k+1} - C_k|}{C_k} < \varepsilon$

3 - $\dfrac{b-a}{2^k} < \varepsilon$

Ex: Find the root of $f(x) = x^2 - 3$, start with $[1, 2]$, $\varepsilon = 0{,}01$ by using bisection method.

$f(a) \cdot f(b) < 0$, works,

Ex $f(x) = x^2 - 3$, $[1, 2]$, root

#	a	b	f(a)	f(b)	$c = \frac{a+b}{2}$	f(c)	update
1.	1	2	-2	1	(1+2)/2=1.5	-0.75	a=c
2.	1.5	2	-0.75	-	(1.5+2)/2=1.75	0.062	b=c
3.	1.5	1.75	-0.75	-	1.625	-0.359	a=c
4.	1.625	1.75	-0.3594		1.6875	-0.1523	a=c
5.	1.6875	1.75	-0.1523		1.7188	-0.0457	a=c
6.	1.7188	1.75	-0.0457		1.7344	0.0081	b=c
7.	1.7188	1.7344	-0.0457		1.7266	-0.0189	

root = 1.7344

Ex. $f(x) = x^3 + 3x - 5$, $[1, 2]$, $\varepsilon = 0.001$ | $f(a) \cdot f(b) < 0$

#	a	b	f(a)	f(b)	c	f(c)
1.	1	2	-1	9	1.5	2.875
2.	1	1.5	-1		1.25	0.70313
3.	1	1.25	-1		1.125	-0.20117
4.	1.125	1.25	-0.201		1.1875	0.23706
5.	1.125	1.1875	-0.201		1.15625	0.014557
6.	1.125	1.15625	-0.201		1.140625	-0.094143
7.	1.140625	1.15625	-0.094		1.148438	-0.040003
8.	1.148438	1.15625	-0.04..		1.152344	-0.012776
9.	1.152344	1.15625	-0.01		1.154297	0.000877