Eger $V = V(x,y,z)$ ise

$$\left.\begin{aligned} E_x &= -\frac{\partial V}{\partial x} \\ E_y &= -\frac{\partial V}{\partial y} \\ E_z &= -\frac{\partial V}{\partial z} \end{aligned}\right] \quad \vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$$

$V(x,y,z) = 3x^2y + y^2 + yz$ ise

$$E_x = -\frac{\partial V}{\partial x} = -6xy$$

$$E_y = \frac{-\partial V}{\partial y} = -3x^2 - 2y - z$$

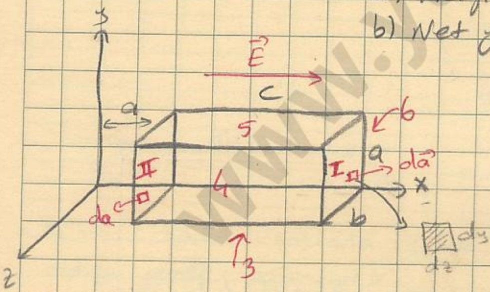
$$E_z = -\frac{\partial V}{\partial z} = -y$$

Örnek Problem Çözümleri

1-) Boyutları $a=b=0,4m$ ve $c=0,6m$ olan kapalı bir yüzey şekildeki gibi yerleştirilmiştir. Bu bölgede elektrik alan $\vec{E} = (3+2x^2)\hat{i}$ N/C olduğuna göre

a) Akıyı hesaplayınız.

b) Net yük miktarı ne kadardır?



$\vec{E}$ düzgün değil x'e bağlı

a) $\Phi = \oint_S \vec{E}\,d\vec{a} = \frac{q_{iç}}{\varepsilon_0}$

$= \oint_1 \vec{E}\,d\vec{a} + \oint_2 \vec{E}\,d\vec{a} + \oint_3 \vec{E}\,d\vec{a} - - - \oint_6 \vec{E}\,d\vec{a}$

$\Phi = 0 \rightarrow \vec{E} \perp d\vec{a}$

$\oint_1 (3+2(a+c)^2)\hat{i}.(dy.dz).\hat{i} + \oint_2 (3+2a^2)\hat{i}.(-dydz)\hat{i}$

$\iint (3+2(a+c)^2)dydz - \iint (3+2a^2)\,dydz$

$3+2(a+c)^2 \underbrace{\int_0^a dy \int_0^b dz}_{a.b} - (3+2a^2). \underbrace{\int_0^a dy \int_0^b dz}_{a.b}$

$(3+2a^2+2c^2+4ac)ab - (3+2a^2)ab$

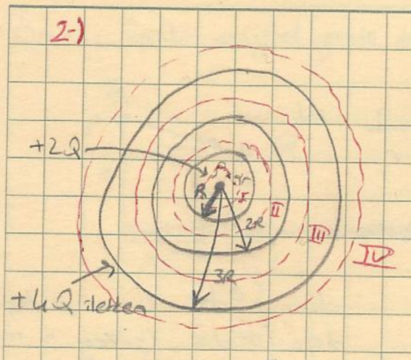
$\Phi \cong 0,27 \ \frac{Nm^2}{C}$

b) $\Phi = \oint \vec{E}\,d\vec{a} = \frac{q_{iç}}{\varepsilon_0}$

$(0,27).\underset{8,85.10^{-12}}{\varepsilon_0} = q_{iç}$

$\boxed{q_{iç} = 2,39.10^{-12}\,C}$

2-)



a) $\alpha = \alpha(\theta, R) = ?$

b) $r < R$

c) $R < r < 2R$

d) $2R < r < 3R$

e) $r > 3R$

a) $q = \int \rho \, dv$

$\downarrow$

$2Q = \int (\alpha r)(4\pi r^2 dr)$

$= 4\pi\alpha \int r^3 dr$

$= 4\pi\alpha \cdot \frac{r^4}{4}\Big|_0^R \rightarrow \cancel{4}\pi\alpha \cdot \frac{R^4}{\cancel{4}}$

$2Q = R^4 \pi \alpha$

$$\boxed{\alpha = \frac{2Q}{R^4 \pi} \quad \frac{C}{m^4}}$$

b) $\oint \vec{E} \, d\vec{a} = \frac{q_{iç}}{\varepsilon_0}$ $\quad \oint E \, da = \frac{q_{iç}}{\varepsilon_0} = E \underbrace{\oint da}_{4\pi r^2}$

$E \cdot 4\pi r^2 = \frac{q_{iç}}{\varepsilon_0}$

$q_{iç} = \pi\alpha r^4 \rightarrow \pi \cdot \frac{2Q}{R^4 \pi} \cdot r^4 \rightarrow q_{iç} = 2Q \frac{r^4}{R^4}$

$E \cdot 4\pi r^2 = \frac{2Qr^4}{R^4 \varepsilon_0}$

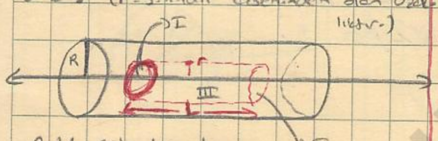
$\vec{E} = \frac{2Q \cancel{r^4}^{r^2}}{4\pi \cancel{r^2} \varepsilon_0 R^4} \rightarrow \frac{2Qr^2}{R^4 4\pi\varepsilon_0} = \boxed{\frac{1}{4\pi\varepsilon_0} \cdot \frac{2Q}{R^4} \cdot r^2 \cdot \hat{r}}$

c) $\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Q}{r^2} \cdot \hat{r}$ $\rightarrow$ yük dağılımının dışı $\rightarrow$ "noktasal yük gibi"

d) $\vec{E} = 0$ $\rightarrow$ iletken içi

e) $\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{6Q}{r^2} \cdot \hat{r}$

3-) Sonsuz uzun R yarıçaplı bir silindir, $\rho = kr$ şeklinde hacimsel yük yoğunluğuna sahiptir. Silindirin içinde ve dışında $\vec{E} = ?$ (r = silindir ekseninden olan uzaklıktır.)



R ≪ L silindir boyu

a) $r < R$

$\oint \vec{E} \, d\vec{a} = \frac{q_{iç}}{\varepsilon_0}$

$\oint_1 \vec{E} \, d\vec{a} + \oint_2 \vec{E} \, d\vec{a} + \oint_3 \vec{E} \, d\vec{a} = \frac{q_{iç}}{\varepsilon_0}$

$\vec{E} \parallel d\vec{a}$, $\theta = 0°$

$E \cdot \underbrace{\oint da}_{yanal} \rightarrow E \cdot 2\pi r L = \frac{q_{iç}}{\varepsilon_0}$

$q_{iç} = \int \rho \, dv \rightarrow \int kr(2\pi r L dr)$

$= k\pi 2L \int_0^r r^2 dr$

$= k 2\pi L \frac{r^3}{3} = q_{iç}$

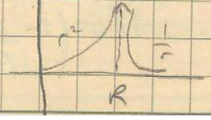
$E 2\pi r L = k 2\pi L \frac{r^3}{3\varepsilon_0}$

$$\boxed{\vec{E} = \frac{kr^2}{3\varepsilon_0} \cdot \hat{r}}$$

b)

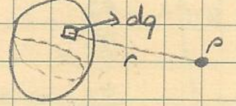
$E \cdot 2\pi r L = k 2\pi L \frac{R^3}{3\varepsilon_0}$

$$\boxed{\vec{E} = \frac{kR^3}{3\varepsilon_0 \cdot r} \cdot \hat{r}}$$



Sürekli Yük Dağılımlarının Potansiyeli

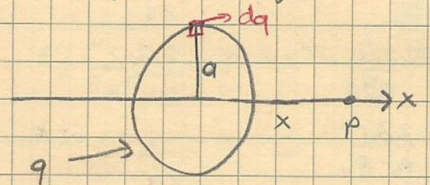
Noktasal bir yükün potansiyeli $= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r}$ idi.



$V = \int \frac{k\,dq}{r}$ ile hesaplanır.

$dq = \int \rho\, dV$ (hacim yük dağılımı)
$dq = \int \sigma\, da$ (yüzey " ")
$dq = \int \lambda\, dl$ (çizgisel " ")

Örnek = Düzgün olarak yüklenmiş bir halkanın potansiyeli



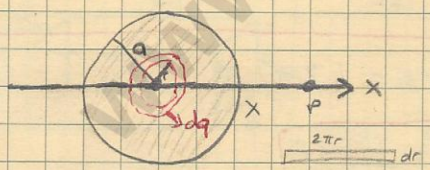
$$V = \int \frac{k\,dq}{r} = \int \frac{k\,dq}{\sqrt{a^2+x^2}} = \frac{k}{\sqrt{a^2+x^2}}\cdot\underbrace{\int dq}_{q}$$

$$V = \frac{kq}{\sqrt{x^2+a^2}}$$

$$E_x = -\frac{dV(x)}{dx} = -kq\frac{d}{dx}(x^2+a^2)^{-\frac{1}{2}} = -kq\left(\frac{-1}{2}\right)(x^2+a^2)^{-\frac{3}{2}}.2x$$

$$\vec{E}_x = \frac{kqx}{(x^2+a^2)^{3/2}}\,\hat{i}$$

Örnek: Düzgün olarak yüklenmiş bir diskin potansiyeli



$$V = \int \frac{k\,dq}{r} = \int \frac{k\,dq}{\sqrt{r^2+x^2}} = \int \frac{k.\sigma.da}{\sqrt{r^2+x^2}}$$

$$= \int \frac{k.\sigma.2\pi r\,dr}{\sqrt{r^2+x^2}} = k\sigma\pi.\int \frac{2r\,dr}{\sqrt{r^2+x^2}}$$

$r^2+x^2=u \qquad 2r\,dr = du$

$$= k\sigma\pi\int\frac{du}{\sqrt{u}} = k\sigma\pi\int u^{-\frac{1}{2}}.du$$

$$= k\sigma\pi\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\Big|_0^a = k\sigma\pi.2.\sqrt{r^2+x^2}\Big|_0^a$$

$dq = \sigma\, da$

$$V(x) = (2\pi k\sigma)(\sqrt{a^2+x^2} - |x|) \qquad x>0 \text{ ise } |x| = x$$

$$V(x) = 2\pi k\sigma(\sqrt{x^2+a^2} - x)$$

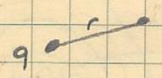
$$E_x = -\frac{dV(x)}{dx} = -2\pi k\sigma\frac{d}{dx}(\sqrt{x^2+a^2} - x)$$

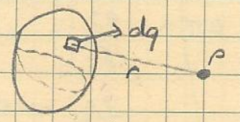
$$E_x = -2\pi k\sigma\left(\frac{1}{2}(x^2+a^2)^{-1/2}.2x - 1\right)$$

$$E_x = 2\pi k\sigma\left(1 - \frac{x}{\sqrt{x^2+a^2}}\right)$$

Sürekli Yük Dağılımlarının Potansiyeli

Noktasal bir yükün potansiyeli $= \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q}{r}$ idi.





$V = \int \dfrac{k\,dq}{r}$ ile hesaplanır.

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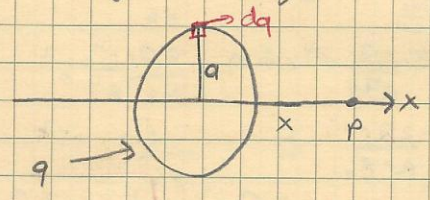
$dq = \int \sigma\, da$ (yüzey " ")
$dq = \int \lambda\, dl$ (çizgisel " ")

Örnek = Düzgün olarak yüklenmiş bir halkanın potansiyeli



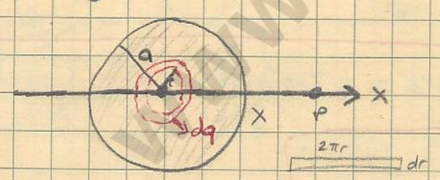
$$V = \int \frac{k\,dq}{r} = \int \frac{k\,dq}{\sqrt{a^2+x^2}} = \frac{k}{\sqrt{a^2+x^2}} \cdot \underbrace{\int dq}_{q}$$

$$V = \frac{kq}{\sqrt{x^2+a^2}}$$

$$E_x = -\frac{dV(x)}{dx} = -kq\frac{d}{dx}(x^2+a^2)^{-\frac{1}{2}} = -kq\left(-\frac{1}{2}\right)(x^2+a^2)^{-\frac{3}{2}} \cdot 2x$$

$$\vec{E}_x = \frac{kqx}{(x^2+a^2)^{3/2}}\ \hat{\imath}$$

Örnek: Düzgün olarak yüklenmiş bir diskin potansiyeli



$dq = \sigma\, da$

$$V = \int \frac{k\,dq}{r} = \int \frac{k\,dq}{\sqrt{r^2+x^2}} = \int \frac{k.\sigma.da}{\sqrt{r^2+x^2}}$$

$$= \int \frac{k.\sigma.2\pi r\,dr}{\sqrt{r^2+x^2}} = k\sigma\pi . \int \frac{2r\,dr}{\sqrt{r^2+x^2}}$$

$r^2+x^2 = u \qquad 2r\,dr = du$

$$= k\sigma\pi \int \frac{du}{\sqrt{u}} = k\sigma\pi \int u^{-\frac{1}{2}}.du$$

$$= k\sigma\pi \frac{u^{\frac{1}{2}}}{\frac{1}{2}}\Big|_0^a = k\sigma\pi.2.\sqrt{r^2+x^2}\Big|_0^a$$

$$V(x) = (2\pi k\sigma)(\sqrt{a^2+x^2} - |x|) \checkmark \qquad x>0 \text{ ise } |x| = x$$

$$V(x) = 2\pi k\sigma(\sqrt{x^2+a^2} - x) \checkmark$$

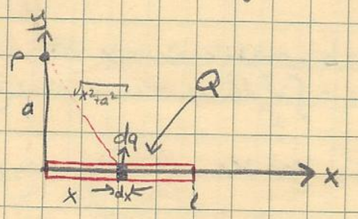
$$E_x = -\frac{dV(x)}{dx} = -2\pi k\sigma\frac{d}{dx}(\sqrt{x^2+a^2} - x)$$

$$E_x = -2\pi k\sigma\left(\frac{1}{2}(x^2+a^2)^{-1/2}.2x - 1\right)$$

$$E_x = 2\pi k\sigma\left(1 - \frac{x}{\sqrt{x^2+a^2}}\right) \checkmark$$

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left(x+\sqrt{x^2+a^2}\right)$$ verilişor

Örnek: Sonlu çizgisel yüklü bir telin elektrik alanı



$$\lambda = \frac{Q}{\ell} \qquad \lambda = \frac{dq}{dx}$$

$$V = \int \frac{k\,dq}{r} = \int_0^{\ell} \frac{k\cdot\lambda\,dx}{\sqrt{x^2+a^2}}$$

$$V = k\lambda \int_0^{\ell} \frac{dx}{\sqrt{x^2+a^2}} = k\lambda \ln\left(x+\sqrt{x^2+a^2}\right)\Big|_0^{\ell}$$

$$V = k\lambda \ln\left(\left(\ell+\sqrt{\ell^2+a^2}\right) - a\right)$$

y ekseni üzerinde yza nokt. potansiyel $V = \frac{kQ}{\ell} \ln\left(\frac{\ell+\sqrt{\ell^2+a^2}}{a}\right)$

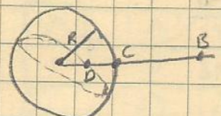
y üzerinde bir yerde $V = \frac{kQ}{\ell} \ln\left(\frac{\ell+\sqrt{\ell^2+y^2}}{y}\right)$

Örnek: Düzgün yüklenmiş bir kürenin potansiyeli

Düzgün dağılmış pozitif yük yoğunluğuna sahip, toplam yükü Q, R yarıçaplı yalıtılmış bir küre veriliyor.

A) Kürenin dışındaki potansiyel? $r>R$

B) Kürenin içindeki potansiyel? $r<R$



A)

$$r>R:\ V_B = -\int_{\infty} \vec{E}\cdot d\vec{l} = -\int_{\infty} \frac{kQ}{r^2}\cdot dr = -kQ\left(\frac{r^{-1}}{-1}\right) = \frac{kQ}{r}\Big|_{\infty}$$

$V_B = \frac{kQ}{r}$ Dolu kürenin dışında potansiyel, noktasal yükün potansiyeline özdeş oldu.

$$\vec{E}_{dış} = \frac{kQ}{r^2}\cdot\hat{r}$$

Yüzeyde $r=R$ $V_C = \frac{kQ}{R}$

$$d\vec{l} = dr\,\hat{r}$$

B)

$$r<R:\ V_D - V_C = -\int_C^D \vec{E}_{iç}\cdot d\vec{l} = -\int_{r=R}^{r=r} \frac{kQr}{R^3}\,dr = -\frac{kQ}{R^3}\left(\frac{r^2}{2}\Big|_R\right)$$

$$\vec{E}_{iç} = \frac{kQ}{R^3}\cdot r\cdot\hat{r}$$

$$V_D - V_C = \frac{kQ}{2R^3}\left(R^2-r^2\right) \rightarrow V_D = \frac{kQ}{2R^3}\left(R^2-r^2\right) + \frac{kQ}{R}$$

$$V_D = \frac{kQR^2}{2R^3} - \frac{kQr^2}{2R^3} + \frac{kQ}{R} \rightarrow \frac{kQ}{2R} - \frac{kQr^2}{2R^3} + \frac{kQ}{R}$$

$$V_D = \frac{kQ}{2R}\left(1 - \frac{r^2}{R^2} + 2\right) = \frac{kQ}{2R}\left(3 - \frac{r^2}{R^2}\right)$$

Yüklü Bir İletkenin Potansiyeli

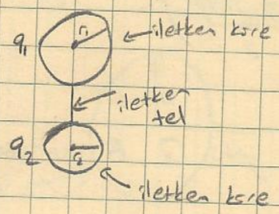
Keyfi a ve b noktaları için

$$V_B - V_A = -\int_A^B \vec{E}_{iç}\, d\vec{l} = 0$$

$$V_B - V_A = 0 \qquad V_B = V_A$$

İletkenin içi ve yüzeyi aynı potansiyele sahiptir. Yani Eşpotansiyellidir.

Örnek: Birbirine bağlı iki iletken küre



$$V_1 = \frac{kq_1}{r_1} \qquad V_2 = \frac{kq_2}{r_2}$$

$$V_1 = V_2 \rightarrow \frac{\cancel{k}q_1}{r_1} = \frac{\cancel{k}q_2}{r_2}$$

$$\boxed{\frac{q_1}{r_1} = \frac{q_2}{r_2}} \qquad r_1 > r_2 \rightarrow q_1 > q_2$$

Yüzeyde elektrik alanları:

$$E_1 = \frac{kq_1}{r_1^2} \qquad E_2 = \frac{kq_2}{r_2^2} \qquad \frac{E_1}{E_2} = \frac{\frac{kq_1}{r_1^2}}{\frac{kq_2}{r_2^2}} = \frac{\cancel{\frac{q_1}{r_1}} \cdot \frac{1}{r_1}}{\cancel{\frac{q_2}{r_2}} \cdot \frac{1}{r_2}} = \boxed{\frac{r_2}{r_1}}$$