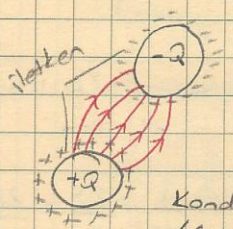
26. Bölüm: Sığa ve Dielektrikler

Sığa: Bir kondansatörün sığası iletkenlerden biri üzerindeki yükün büyüklüğünün, bunlar arasındaki potansiyel farka oranıdır.



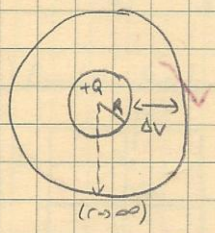
Kondansatör (Sığaç)

Sığa : C $\quad [C] = \frac{\text{Coulomb}}{\text{Volt}} = \text{Farad}$

$$C = \frac{Q}{\Delta V}$$

Sığa Hesabı

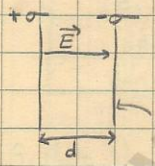
1-) Zıt işaretli yükler ile yüklenmiş iki küre göz önüne alalım. Bunlardan birinin yarıçapı R, diğeri çok büyük (sonsuz) olsun.



$$\Delta V = -\int_{\infty}^{R} \vec{E}\,d\vec{l} = \frac{kQ}{R} \rightarrow C = \frac{Q}{\Delta V} = \frac{R}{k} = \frac{R}{\frac{1}{4\pi\varepsilon_0}}$$

$$C = 4\pi\varepsilon_0 R$$

Paralel plakalı kondansatör



$E = \frac{\sigma}{\varepsilon_0}$ idi $\rightarrow$ $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$

$$\Delta V = Ed = \frac{Qd}{\varepsilon_0 A} \rightarrow \frac{Q}{\Delta V} = C = \frac{\varepsilon_0 A}{d}$$

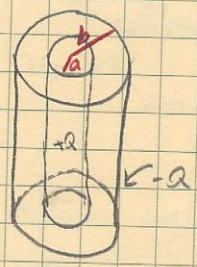
$\sigma = \frac{Q}{A}$

$$\boxed{C = \frac{\varepsilon_0 A}{d}}$$

2- Silindirik Kondansatör

Dolu bir silindirin yarıçapı a, yükü $+Q$ 'dur. Aynı eksenli daha büyük kalınlığı ihmal edilebilen bir kabuğun yarıçapı b ve yükü $-Q$ 'dur. ℓ uzunluklu bir silindirik kondansatörün sığası?

(Çok uzun silindir için hesap yapılır).



$$V_b - V_a = -\int_a^b \vec{E}\,d\vec{\ell}$$

$$= -2k\lambda \int_a^b \frac{dr}{r}$$

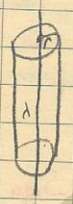
$$= -2k\lambda \ln(r)\Big|_a^b$$

$$= -2k\lambda \ln\left(\frac{b}{a}\right), \quad \lambda = \frac{Q}{\ell}$$

$$\Delta V = -2k\frac{Q}{\ell}\ln\left(\frac{b}{a}\right) \rightarrow |\Delta V| = 2k\frac{Q}{\ell}\ln\left(\frac{b}{a}\right)$$

$$C = \frac{Q}{\Delta V} = \frac{\ell}{2k} \cdot \frac{1}{\ln(b/a)} = \frac{\ell}{2k\ln(b/a)}$$ ✓

Daha önce $+\lambda$ yüklü uzun tel için

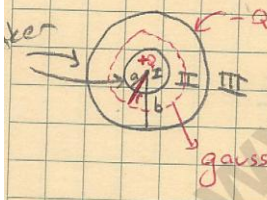


$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2\lambda}{r} = \frac{2k\lambda}{r}$$

idi.

Birim uzunluk başına sığa $= \frac{C}{\ell} = \frac{1}{2k\ln(b/a)}$, $\frac{\text{Farad}}{\text{m}}$

3- Küresel Kondansatör



I. ve III. bölgede $\vec{E} = 0$

$$V(b) - V(a) = -\int_a^b \vec{E}\,d\vec{\ell}, \quad d\vec{\ell} = dr\,\hat{r}$$

$$\Delta V = -\int_a^b \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2}\,dr$$

$$= -\frac{Q}{4\pi\varepsilon_0}\cdot\int_a^b r^{-2}dr = -\frac{Q}{4\pi\varepsilon_0}\cdot\left(\frac{r^{-1}}{-1}\Big|_a^b\right)$$

$$= \frac{Q}{4\pi\varepsilon_0}\cdot\frac{1}{r}\Big|_a^b \rightarrow \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{b}-\frac{1}{a}\right) \quad b>a$$

$$\Delta V = -\frac{Q}{4\pi\varepsilon_0}\cdot\left(\frac{b-a}{ab}\right) \quad b>a$$

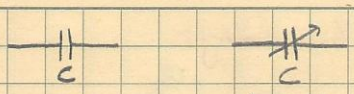
$$|\Delta V| = \frac{Q}{4\pi\varepsilon_0}\cdot\left(\frac{b-a}{ab}\right)$$

$$\frac{Q}{\Delta V} = C = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$$ ✓

$$\oint \vec{E}\,d\vec{A} = \frac{Q}{\varepsilon_0}$$

$$E\cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

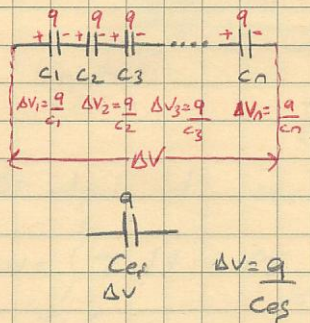
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2}\cdot\hat{r}$$ ✓



kondansatör Ayarlı kondansatör

Kondansatörlerin Bağlanması

a) Seri bağlama



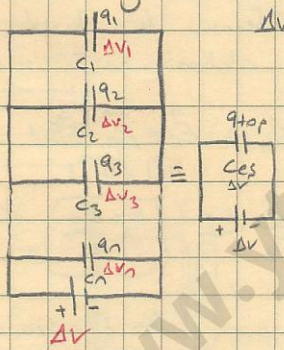
$\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3 \dots + \Delta V_n$

$\frac{q}{C_{es}} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3} \dots \frac{q}{C_n}$

$$\frac{1}{C_{es}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \dots \frac{1}{C_n}$$

$\Delta V = \frac{q}{C_{es}}$

b) Paralel bağlama



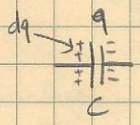
$\Delta V_1 = \Delta V_2 = \Delta V_3 = \Delta V_n = \Delta V$

$q_{top} = q_1 + q_2 + q_3 \dots q_n$

$C_{es}\Delta V = C_1\Delta V_1 + C_2\Delta V_2 + C_3\Delta V_3 \dots C_n\Delta V_n$

$$C_{es} = C_1 + C_2 + C_3 \dots C_n$$

Yüklü Kondansatörde Depolanan Enerji



$W = qV$ idi.

$\int dW = \int dq \cdot \frac{q}{C}$

$$W = \int_{q=0}^{Q} \frac{1}{C} q\,dq$$

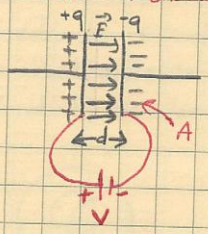
$$W = \frac{1}{2}\cdot\frac{Q^2}{C} = \frac{Q^2}{2C}$$

$Q = CV$ yazılır

$$W = \frac{C^2V^2}{2C} = \frac{1}{2}CV^2$$

(J)

Düzlem Kondansatörde Birim Hacimde Depolanan Enerji



$E = \frac{V}{d}$

$U = \frac{1}{2} C V^2$

$= \frac{1}{2} \left(\frac{\varepsilon_0 A}{d}\right)(Ed)^2$

$= \frac{1}{2} \varepsilon_0 (Ad) E^2$

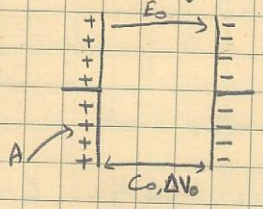
↓ levhalar arasındaki hacim V

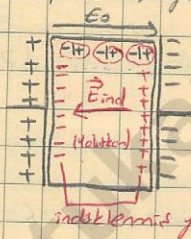
$u = \frac{U}{V} = \frac{1}{2} \varepsilon_0 E^2, \ \frac{J}{m^3}$

↓ enerji yoğunluğu

Dielektrikli Kondansatör

Dielektrik = yalıtkan (porselen, teflon, plastik, yağlı kağıt ...)





Uçlanmış bir kondansatöre ek alalım.

$\vec{E}_{net} = \vec{E} = \vec{E_0} - \vec{E}_{ind}$

$|\vec{E}| < |\vec{E_0}|$

$\frac{E}{E_0} < 1$ $\quad \boxed{E = \frac{E_0}{K}}$ $\quad K > 1$

K = (kappa): Dielektrik sabiti

$\boxed{K = \frac{\varepsilon}{\varepsilon_0}}$ ✓

$\Delta V = \frac{\Delta V_0}{K} \Rightarrow \Delta V_0 = K \cdot \Delta V$

$\frac{Q}{C_0} = K \cdot \frac{Q}{C}$

$\boxed{C = K \cdot C_0}$ ✓

Enerji ↓

↳ Kaynak enerji elektriğin iyeri çekilmesi sırasında harcanır.

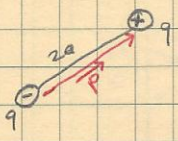
$U_0 = \frac{1}{2} C_0 \Delta V_0^2$ veya

$U_0 = \frac{Q^2}{2C_0} \rightarrow U = \frac{Q^2}{2C} = \frac{Q^2}{2KC_0}$

$\boxed{U = \frac{U_0}{K}}$

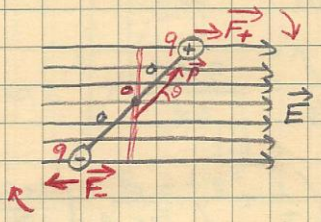
$\frac{1}{K}\left(\frac{Q^2}{2C_0}\right) \rightarrow U_0$

Bir Elektrik Alandaki Elektrik Dipolü



$\vec{P}$ = dipol moment vektörü

$P = 2aq$



Tork: $\tau = 2.F.a.\sin\vartheta$ $\qquad \vec{\tau} = \vec{P}.\vec{E}$

$F = q.E$

$\tau = 2.q.E.a.\sin\vartheta$

$= 2aqE\sin\vartheta$

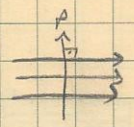
$\boxed{\tau = PE\sin\vartheta}$ ✓

Bir dipolü $d\vartheta$ kadar döndürmek için yapılan iş: $dW = \tau d\vartheta$

$$W = \int_{\vartheta_i}^{\vartheta_s} \tau d\vartheta \Rightarrow PE \int_{\vartheta_i}^{\vartheta_s} \sin\vartheta . d\vartheta = PE(-\cos\vartheta)\Big|_{\vartheta_i}^{\vartheta_s}$$

$$W = -PE(\cos\vartheta_s - \cos\vartheta_i)$$

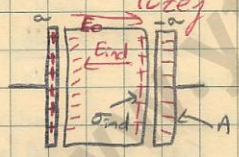
$\vartheta_i = \pi/2 = 90°$ olsun ($\cos 90° = 0$)



$$W = -PE\cos\vartheta$$

$$\boxed{W = -\vec{P}\vec{E}} \Rightarrow U = -\vec{P}\vec{E}$$

Yüzey Yük Yoğunlukları



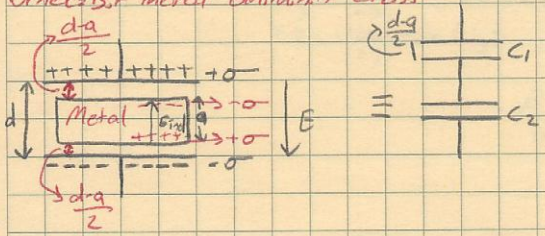
$$\frac{\sigma}{K\varepsilon_0} = \frac{\sigma}{\varepsilon_0} - \frac{\sigma_{ind}}{\varepsilon_0}$$

$E = E_0 - E_{ind}$

$\vec{E} = \frac{\vec{E}_0}{K}$

$$\boxed{\sigma_{ind} = \left(\frac{K-1}{K}\right)\sigma}$$ ✓

Örnek: Bir metal dilimin etkisi:



$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{\frac{\varepsilon_0 A}{\frac{(d-a)}{2}}} + \frac{1}{\frac{\varepsilon_0 A}{\frac{(d-a)}{2}}}$$

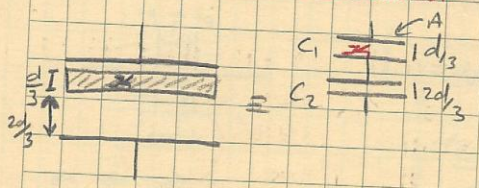
$$\boxed{C = \frac{\varepsilon_0 A}{d-a}}$$

i) metal dilim çok ince: $a \ll d \Rightarrow d-a \approx d$

ii) kalın : $a \rightarrow d$

yokken $\Rightarrow C_0 = \frac{\varepsilon_0 A}{d}$ //

Örnek: Kısmen dolu kondansatör



$C_{es} = ?$

$$C_1 = K\frac{\varepsilon_0 A}{d/3} = \frac{3K\varepsilon_0 A}{d}$$

$$C_2 = \frac{\varepsilon_0 A}{2d/3} = \frac{3\varepsilon_0 A}{2d}$$

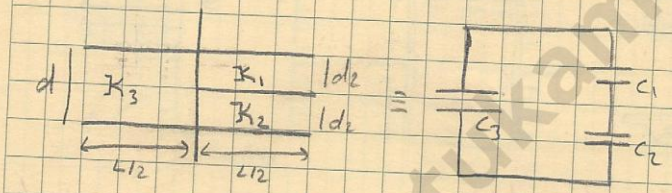
$$\frac{1}{C_{es}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d}{3K\varepsilon_0 A} + \frac{2d}{3\varepsilon_0 A} = \frac{d}{3\varepsilon_0 A}\left(\frac{1}{K} + 2\right) = \frac{d}{3\varepsilon_0 A}\left(\frac{1+2K}{K}\right)$$

$$C_{es} = \left(\frac{3K}{1+2K}\right)\left(\frac{\varepsilon_0 A}{d}\right)$$

$\sim$ A kesitli d mesafeli ε_0 boşluk

$$C_{es} = \underbrace{\left(\frac{3K}{1+2K}\right)}_{>1} C_0$$

Sığa Artar



$$C_1 = K_1\varepsilon_0\frac{A/2}{d/2} = K_1\underbrace{\left(\frac{\varepsilon_0 A}{d}\right)}_{C_0} = K_1 C_0$$

$$C_2 = K_2\varepsilon_0\frac{A/2}{d/2} = K_2 C_0$$

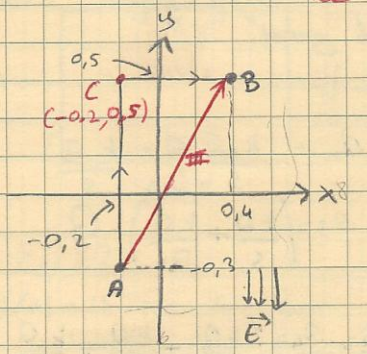
$$C_3 = K_3\varepsilon_0\frac{A/2}{d} = \frac{K_3 C_0}{2}$$

$$\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C_{es} = C_3 + C_{12}$$

Örnek Problemler

1-



Şekildeki gibi düzgün bir elektrik alanı negatif y doğrultusunda ve 325 V/m şiddetinde. A noktasının koordinatları (-0,2, -0,3)m ve B noktasının (0,4, 0,5)m 'dir. $V_B - V_A$?

$\vec{E} = (-325\,\hat{j}\ V/m)$

$$V_B - V_A = -\int_A^B \vec{E}\,d\vec{\ell} = -\int_A^C \vec{E}\,d\vec{\ell} - \int_C^B \vec{E}\,d\vec{\ell}$$

$d\vec{\ell} = dy\,\hat{j}$ $\qquad$ $d\vec{\ell} = dx\,\hat{i}$

$$V_B - V_A = -\int_A^C (-325\,\hat{j}).dy.\hat{j} - \int_C^B (-325\,\hat{j}).dx.\hat{i}$$

$\hat{j}.\hat{j} = 1 \to \cos\theta = 1$ ($\theta = 0°$) $\qquad$ $\hat{j}.\hat{i} = 0 \to \cos\theta = 0$ ($\theta = 70°$)

$$-\int_A^C (-325)\,dy \to 325.y\Big|_{-0,3}^{0,5} \to 325.0,8 = \underline{260\ volt}$$

Ek: III. yoldan: $\vec{E} = (-325\,\hat{j})\ V/m$

$$V_B - V_A = -\int_A^B \vec{E}\,d\vec{\ell} \to -\int_A^B (-325)\,\hat{j}.(0,6.\hat{i} + 0,8\,\hat{j}) \to -\int_A^B (-325).(0,8)$$

$d\ell\cos\theta + d\ell\sin\theta$ ($\cos\theta = 0$, $\sin\theta = 1$)

2- Uzayın belirli bir bölgesinde elektriksel potansiyel $V = 5x - 3x^2y + 2yz^2$ $325.0,8 = 260$ Volt olarak veriliyor. Elektrik alan vektörünü bulunuz ve $P = (1, 0, -2)$ noktasındaki şiddetini (büyüklüğünü) elde ediniz.

$V = V(x, y, z) \to$

$E_x = -\frac{\partial V}{\partial x} \to -(5 - 6xy) = 6xy - 5$

$E_y = -\frac{\partial V}{\partial y} \to -(-3x^2 + 2z^2) = 3x^2 - 2z^2$

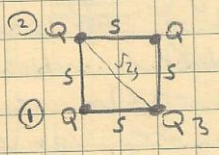
$E_z = -\frac{\partial V}{\partial y} \to -4yz = -4yz$

$\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$

$= (6xy - 5)\hat{i} + (3x^2 - 2z^2)\hat{j} - 4yz\hat{k}$ ✓

$$\vec{E}\Big|_{(P)} = \vec{E}\Big|_{(1,0,-2)} = -5\hat{i} - 5\hat{j} + 0\hat{k} \to -5\hat{i} - 5\hat{j}$$ ✓

3- Bir kenarı "s" olan bir karenin köşelerine "Q" büyüklüğünde özdeş yükler yerleştirmek için ne kadarlık iş yapmak gerekir?



$U_1 = "0"$

$U_2 = V_1 \cdot Q_2 = \frac{k \cdot Q}{s} \cdot Q = \frac{kQ^2}{s}$

$U_3 = V_{12} \cdot Q_3 = \left(\frac{kQ}{s} + \frac{kQ}{\sqrt{2}s}\right) \cdot Q$

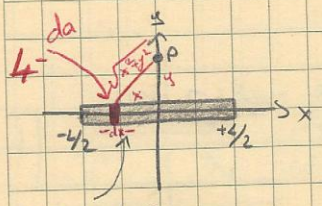
$U_4 = V_{123} \cdot Q_4 = \left(\frac{kQ}{\sqrt{2}s} + \frac{kQ}{s} + \frac{kQ}{s}\right) \cdot Q$

Toplam İşe W_s $4 \cdot \frac{kQ^2}{s} + 2\frac{kQ^2}{\sqrt{2}s} = \frac{kQ^2}{s}(4+\sqrt{2})$ ✓

$\downarrow$

5,41

$5,41 \cdot \frac{kQ^2}{s}$ ✓

4- 

toplam yük Q

$\lambda = \frac{Q}{L} = sbt$

$V_{(p)} = ?$

$V = \int \frac{k\,dq}{r}$ genel formül

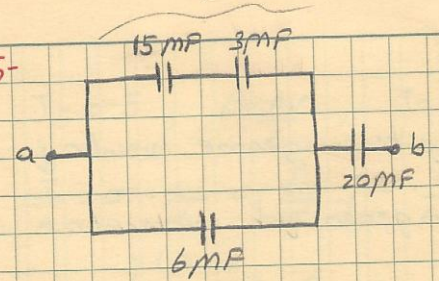
$V = \int \frac{k\,dq}{r} = \int_{-L/2}^{L/2} \frac{k\lambda\,dx}{\sqrt{x^2+y^2}} = k\lambda \int_{-L/2}^{L/2} \frac{dx}{\sqrt{x^2+y^2}}$

$V = k\lambda \cdot \ln\left(x + \sqrt{x^2+y^2}\right)\Big|_{-L/2}^{L/2}$

$V = k\lambda \left(\ln\left(L/2 + \sqrt{(L/2)^2+y^2}\right) - \left(-L/2 + \sqrt{(-L/2)^2+y^2}\right)\right)$

$= \frac{kQ}{L} \ln \frac{\left(L/2 + \sqrt{(L/2)^2+y^2}\right)}{\left(-L/2 + \sqrt{(-L/2)^2+y^2}\right)} = V_{(y)}$

5-



a) C_{es} = ?

b) V_{ab} = 15 ise her bir kondansatörün yükü = ?

a) → 15 ve 3 seri

$$\frac{1}{C_{12}} = \frac{1}{15} + \frac{1}{3} = \frac{6}{15} \qquad C_{12} = \frac{5}{2}\ mF$$

b) $5/2$ ve 6 paralel

$$C_{123} = 5/2 + 6 = \frac{17}{2}\ mF$$

c) $17/2$ ve 20 seri

$$\frac{1}{C_{es}} = \frac{2}{17} + \frac{1}{20} = \frac{57}{340}$$

$$C_{es} = 340/57\ mF = \frac{340}{57} \cdot 10^{-6}\ F$$ ✓

b1 = a — (C = 340/57) — b, V_{ab} = 15 V

$$Q = CV = \frac{340}{57} \cdot 15 = \frac{1700}{19} C$$