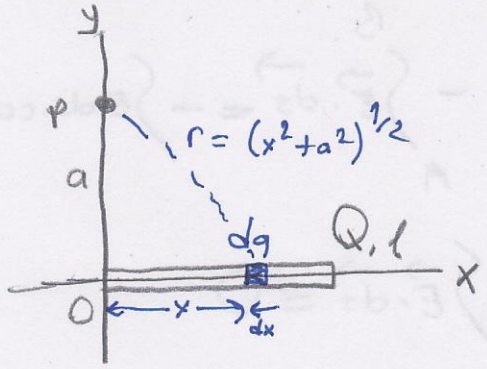
Örnek: Sonlu Çizgisel Yükün Potansiyeli



$$dV = k\frac{dq}{r}$$

$$V_p = \int dV = k\int \frac{dq}{r} = k\int \frac{(\lambda\, dx)}{(x^2+a^2)^{1/2}} =$$

$$= k\cdot\lambda \int \frac{dx}{(x^2+a^2)^{1/2}} \qquad \left(\frac{Q}{\ell}\right)$$

$$\sqrt{x^2+a^2} = u - x \quad \text{dönüşümü}$$

$$x^2+a^2 = (u-x)^2$$

$$x^2+a^2 = u^2 - 2ux + x^2$$

$$x = \frac{u^2-a^2}{2u}$$

$$\frac{dx}{du} = \frac{(2u)(2u) - (u^2-a^2)\cdot 2}{4u^2}$$

$$dx = \frac{u^2+a^2}{2u^2}\,du$$

$$= k\cdot\left(\frac{Q}{\ell}\right)\int \frac{\left(\frac{u^2+a^2}{2u^2}\,du\right)}{u - \underbrace{\left(\frac{u^2-a^2}{2u}\right)}_{x}}$$

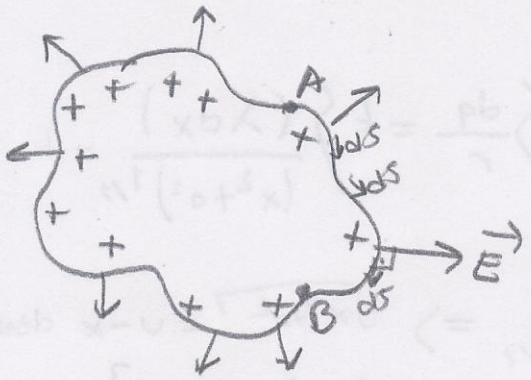
$$= k\cdot\frac{Q}{\ell}\int \frac{\frac{u^2+a^2}{2u^2}\,du}{\underbrace{\frac{2u^2-u^2+a^2}{2u}}_{u^2+a^2}} = k\cdot\frac{Q}{\ell}\int \frac{du}{u} = k\cdot\frac{Q}{\ell}[\ln u]$$

$$= k\cdot\frac{Q}{\ell}\left[\ln\left(\sqrt{x^2+a^2}+x\right)\right]_0^{\ell}$$

$$= k\cdot\frac{Q}{\ell}\left\{\ln\left(\sqrt{\ell^2+a^2}+\ell\right) - \ln a\right\}$$

$$V_p = k\cdot\frac{Q}{\ell}\ln\left(\frac{\sqrt{\ell^2+a^2}+\ell}{a}\right)$$

Yüklü Bir İletkenin Potansiyeli

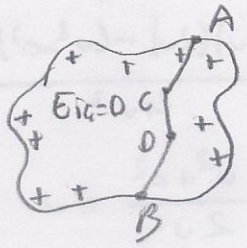


$$\Delta V = V_B - V_A = -\int_A^B \vec{E}\cdot\vec{ds} = -\int E.ds.\cos\theta$$

$$\Delta V = V_B - V_A = -\int \vec{E}\cdot\vec{ds} = 0$$

$$E = -\frac{dV}{ds} = 0 \qquad V = \text{sabit}$$

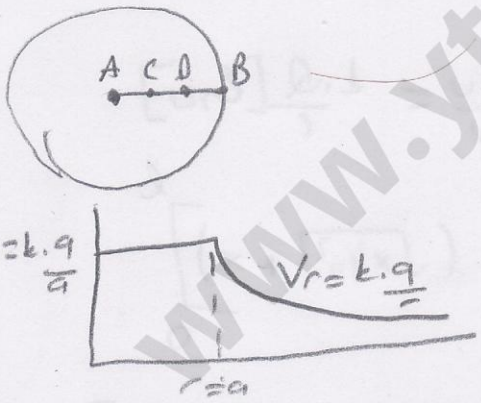
$$V_A = V_B = V_C = V_D \ldots \text{(sabit)}$$

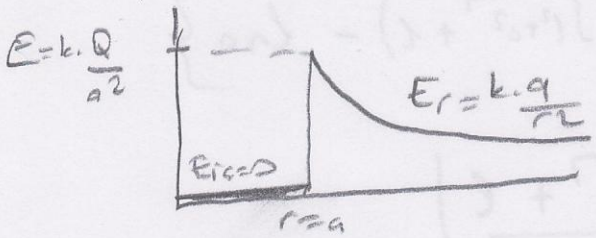


$$\Delta V = V_B - V_A = -\int \vec{E}\cdot\vec{ds} = 0$$

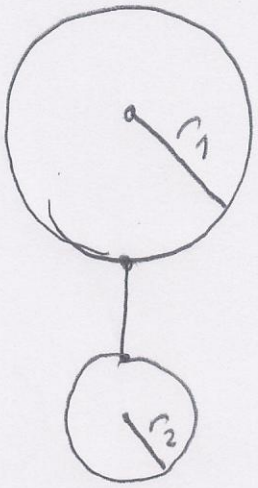
$$\Delta V = 0$$

$$V_A = V_B = V_C$$





Birbirine Bağlı İki Yüklü Küre



Yüklü iki küre ince telle bağlansın birbirine.

Yük akışı $\Delta V = V_2 - V_1 = 0$ $V_1 = V_2$ oluncaya kadar sürer. (denge durumu)

Denge durumunda yükleri q_1 ve q_2 olsun, küre yüzeyinde elektrik alanları oranı?

$$V_1 = k\cdot\frac{q_1}{r_1}$$

$$V_2 = k\cdot\frac{q_2}{r_2}$$

$$\Delta V = V_2 - V_1 = 0$$

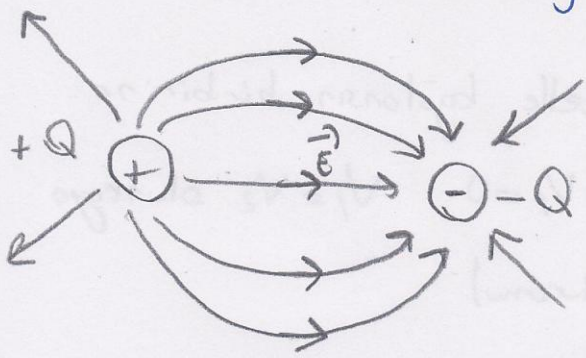
$$V_1 = V_2$$

$$k\cdot\frac{q_1}{r_1} = k\cdot\frac{q_2}{r_2} \rightarrow \boxed{\frac{q_1}{q_2} = \frac{r_1}{r_2}}$$

$$\frac{E_1}{E_2} = \frac{k\cdot\frac{q_1}{r_1^2}}{k\cdot\frac{q_2}{r_2^2}} = \left(\frac{q_1}{q_2}\right)\cdot\left(\frac{r_2^2}{r_1^2}\right) = \left(\frac{r_1}{r_2}\right)\left(\frac{r_2^2}{r_1^2}\right)$$

$$\boxed{\frac{E_1}{E_2} = \frac{r_2}{r_1}}$$

Sığa ve Dielektrik

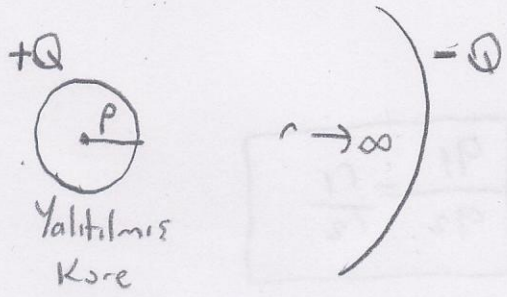


$$C = \frac{Q}{\Delta V}$$

$1F = 1\frac{C}{V}$ (farad)

Sığanın Hesaplanması

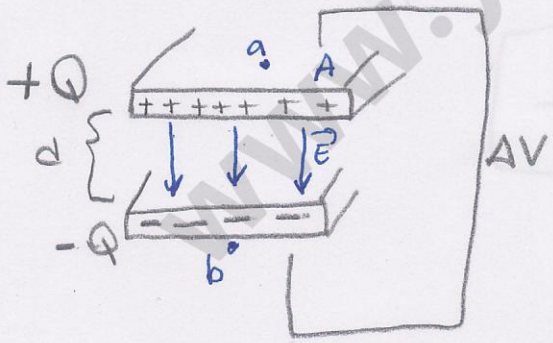
① Yalıtılmış Bir Kürenin Sığası;



$$C = \frac{Q}{\Delta V} = \frac{Q}{V_R - V_\infty} = \frac{Q}{V_R}$$

$$= \frac{Q}{\left(k\frac{Q}{R}\right)} = \frac{R}{k} = \underline{4\pi\varepsilon_0 R}$$

② Paralel Plakalı Kondansatör



iletken plakalar

$$\Delta V = V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{s} = -E\int_a^b ds$$

$$\Delta V = V_b - V_a = -E\cdot d$$

$V_b < V_a$

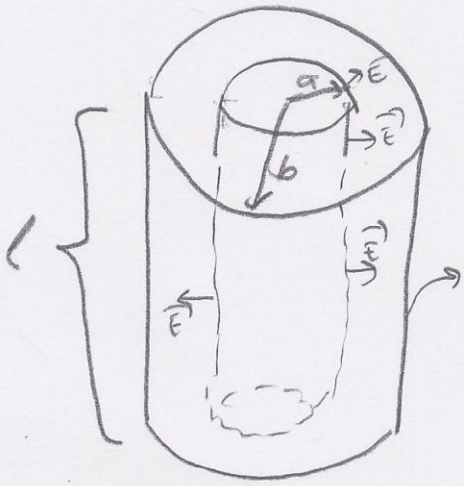
$$E = \frac{\sigma}{\varepsilon}$$ → idi, iletkenin çok yakınında elektrik alan

$$\rightarrow \Delta V = E\cdot d = \left(\frac{\sigma}{\varepsilon_0}\right)d = \left[\frac{Q/A}{\varepsilon_0}\cdot d\right] = \frac{Q}{\varepsilon_0 A}\cdot d$$

Yüzeysel Yük Yoğunluğu $= \sigma = \frac{Q}{A}$

$$C = \frac{Q}{\Delta V} = \varepsilon_0\frac{A}{d}$$

(3) Silindirik Kondansatörün Sığası



$$C = \frac{Q}{\Delta V} \qquad \Delta V = -\int_a^b \vec{E}\cdot d\vec{s}$$

$$\lambda = \frac{q_{iç}}{\ell} = \frac{Q}{\ell}$$

$$\Phi_E = \oint \vec{E}\cdot d\vec{A} = \frac{q_{iç}}{\varepsilon_0}$$

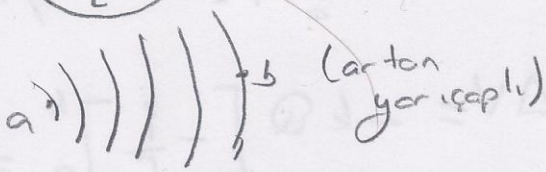
$$E\oint dA = \frac{q_{iç}}{\varepsilon_0}$$

$$E(2\pi r\ell) = \frac{(\lambda\cdot\ell)}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}\cdot\frac{2}{2} = 2k\frac{\lambda}{r}$$

$$\Delta V = -\int_a^b \vec{E}\cdot d\vec{s} = -\int_a^b E\cdot ds\cdot\cos 0^\circ$$

a'dan b'ye
+'dan -'ye



$ds = +dr$

$$= -\int_a^b E\,dr$$

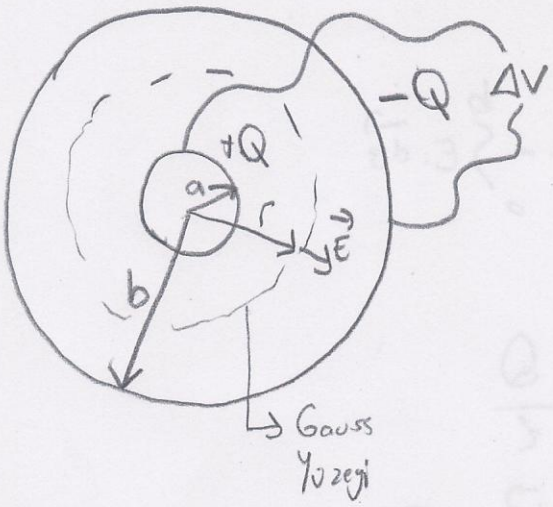
$$= -\int_a^b \left(2k\frac{\lambda}{r}\right)dr$$

$$= -2k\lambda\int_a^b \frac{dr}{r} = [\ln r]_a^b = \ln b - \ln a = \ln\frac{b}{a}$$

$$\Delta V = 2k\frac{Q}{\ell}\left[\ln\frac{b}{a}\right]$$

$$C = \frac{Q}{\Delta V} = \frac{\ell}{2k\ln\left(\frac{b}{a}\right)}\ (F) \qquad \frac{C}{\ell} = \frac{1}{2k\ln\left(\frac{b}{a}\right)}$$

4) Küresel Kondansatörün Sığası



$q_{iç} = Q$

$$\Phi = \oint \vec{E}.d\vec{A} = \frac{q_{iç}}{\varepsilon_0}$$

$$E \oint dA = \frac{q_{iç}}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi r \varepsilon_0 r^2} = k.\frac{Q}{r^2}$$

$$\Delta V = V_b - V_a = -\int_a^b \vec{E}.d\vec{r} = -\int E.ds.\cos\theta$$

$\oplus \rightarrow \ominus$, $\theta = 0°$

$ds = +dr$ artan yarıçap

$$= -\int E.dr = -\int \left(k.\frac{Q}{r^2}\right) dr$$

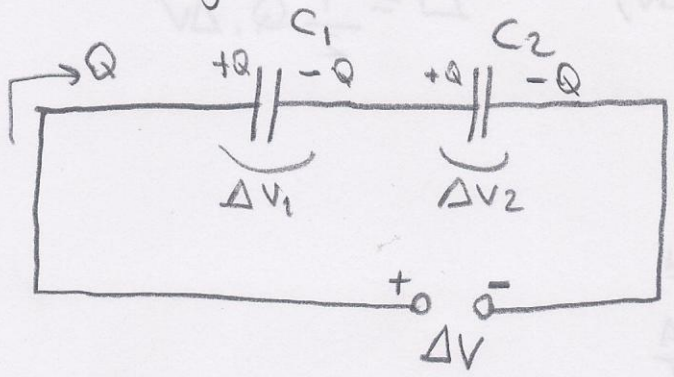
$$= -k.Q \int_a^b \frac{dr}{r^2} = \int r^{-2} dr = \left[\frac{r^{-2+1}}{-2+1}\right] = \left[-\frac{1}{r}\right]_a^b$$

$$\Delta V = -kQ\left[-\frac{1}{r}\right]_a^b = k.Q\left[\frac{1}{b} - \frac{1}{a}\right] = k.Q\left[\frac{a-b}{ab}\right]$$

$$\boxed{C = \frac{Q}{\Delta V} = \frac{a.b}{k.(a-b)}}$$

Kondansatörlerin Bağlanması

① Seri Bağlama:



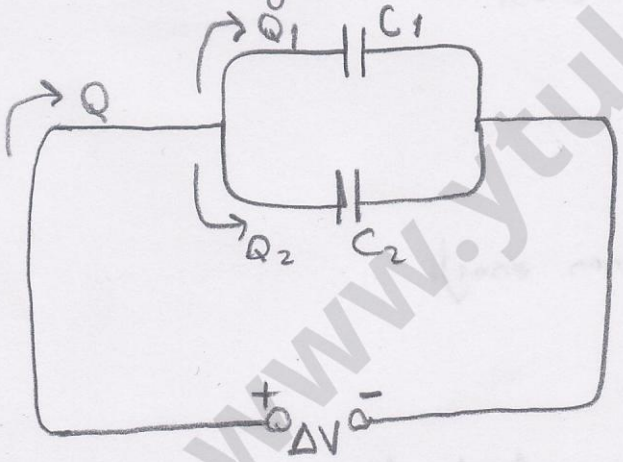
$Q = Q_1 = Q_2 \qquad C = \frac{Q}{\Delta V} \qquad \Delta V = \frac{Q}{C}$

$\Delta V = \Delta V_1 + \Delta V_2$

$$\frac{Q}{C_{esd}} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

$$\frac{1}{C_{es}} = \frac{1}{C_1} + \frac{1}{C_2}$$

② Paralel Bağlama;



$\Delta V = \Delta V_1 = \Delta V_2$

$C = \frac{Q}{\Delta V} \Rightarrow Q = C.\Delta V$

$Q = Q_1 + Q_2$

$C_{esd} \cdot \Delta V = C_1 \cdot \Delta V_1 + C_2 . \Delta V_2$

$C_{esd} = C_1 + C_2$

Yüklü Kondansatörlerde Depolanan Enerji:

$$W = \int f . ds = \int (q.E)\, ds = q \int E . ds = q . \Delta V$$

$dw = dq . \Delta V = dq . \left(\frac{q}{C}\right)$

$C = \frac{q}{\Delta V}$ → Herhangi bir anda kondansatörde biriken yük.

$\Delta V = \frac{q}{C}$

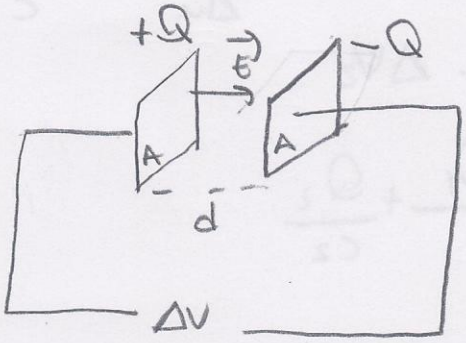
$$W = \int dw = \int \left(\frac{q}{C}\right) . dq = \frac{1}{C} \int_0^Q q . dq$$

$$\boxed{W = \frac{1}{2} \frac{Q^2}{C}}$$ → Kondansatörün yüklenmesinde yapılan iş.

Yüklü Bir Kondansatörde Depolanan Enerji:

$$U = \frac{1}{2}\frac{Q^2}{C}$$

$$U = \frac{1}{2} C (\Delta V)^2 \qquad U = \frac{1}{2} Q \cdot \Delta V$$



$$\Delta V = E \cdot d$$

$$C = \varepsilon_0 \frac{A}{d}$$

$$U = \frac{1}{2} C (\Delta V)^2 = \frac{1}{2}\left(\varepsilon_0 \frac{A}{d}\right)\cdot (E^2 \cdot d^2)$$

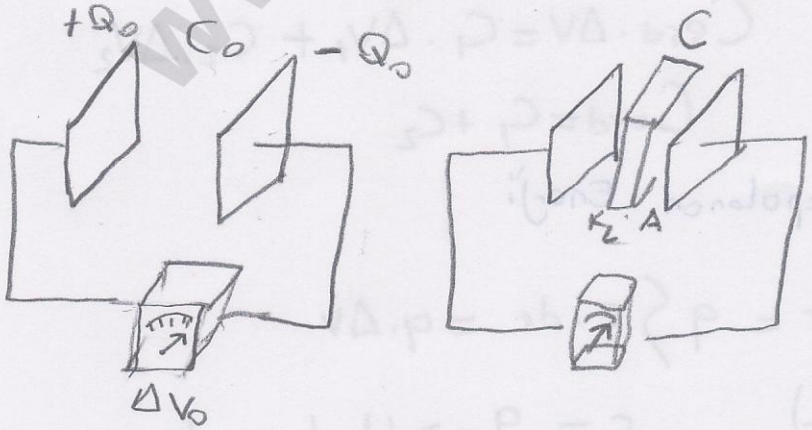
$$U = \frac{1}{2}\varepsilon_0 \cdot E^2 \underbrace{(A \cdot d)}_{\text{plakalar arası}} = \frac{1}{2}\varepsilon_0 \cdot E^2 V$$

$$U_E = \frac{U}{V} = \frac{1}{2}\varepsilon_0 \cdot E^2 \quad (J/m^3)$$

↳ Enerji yoğunluğu

Kondansatörün birim hacmi başına depolanan enerji

Dielektrikli Kondansatörler



$$\Delta V = \frac{\Delta V_0}{K}$$

$$C_0 = \frac{Q_0}{\Delta V_0} \qquad C = \frac{Q_0}{\Delta V} = \frac{Q_0}{\Delta V_0 / K} = K\frac{Q_0}{\Delta V_0}$$

$$C = K \cdot C_0$$

$$U = \frac{1}{2}\frac{Q_0^2}{C} = \frac{1}{2}\cdot\frac{Q_0^2}{KC_0}$$

$$= \frac{1}{K}\left(\frac{1}{2}\frac{Q_0^2}{C_0}\right)$$

$$U = \frac{U_0}{K}$$

$K \rightarrow$ Dielektrik Sabiti

$K \geq 1$