Convergence

$0 \leq g'(x) < 1 \quad x_0 = 1,5$

$g(x) = x^3 - 3 \quad g'(x) = 3x^2 > 1 \quad \times$

$g(x) = \sqrt[3]{x-3} \quad g'(x) = \dfrac{1}{3\sqrt{x+3}} < 1 \quad \checkmark$

$g(x) = \dfrac{3}{x^2-1} \quad g'(x) = \dfrac{6x}{(x^2-1)^2} > 1 \quad \times$

Correct $g(x) = \sqrt[3]{x+3}$

root $= 1,672$

Ex

$f(x) = x - \frac{1}{2}\cos x \quad x_0 = 0,5$ find the root by using fixed-point method $\varepsilon = 0,001$

i) $g(x) = \frac{1}{2}\cos x \qquad$ ii) $g(x) = \arccos(2x)$

$g'(x) = \dfrac{-2}{\sqrt{1-4x^2}} \quad \times$

	x_p	$f(x_p)$
0	0,5	$6,121 \times 10^{-3}$
1	0,4388	$1,384 \cdot 10^{-2}$
2	0,4526	⋮
3	0,4496	
4	0,4503	
5	0,4502	

$\frac{1}{2}\cos(0,5) = 0,4388$

$\frac{1}{2}\cos(0,4388) = 0,4526$

Root $= 0,4502$

5) Secant Method

The secant method is a technique for findinp the root of a scalar-valued function f(x) of a single variable x when no information about deriative

Given a curve, a secant line (or just secant) is a line which passes through two points of that curve.

We have two approximations $\overset{(a)}{x_0}$ and $\overset{(b)}{x_1}$ to a root $\overset{(c)}{r}$ of $f(x)$, that is $f(r)=0$

The secant method is defined by the recorrence relation. Starting with initial values x_0 and x_1, we construct a line through the point $(x_0, f(x_0))$ and $(x_1, f(x_1))$. This line has the equation.

$$y = \frac{f(x_1)-f(x_0)}{x_1-x_0}(x-x_1) + f(x_1)$$

The solution is

$$x = x_1 - f(x_1)\frac{x_1-x_0}{f(x_1)-f(x_0)}$$

Newton

$$x_{p+1} = x_p - \frac{f(x_p)}{f'(x_i)}$$

$$f'(x) = \frac{f(x_i-1)-f(x_i)}{x_i-1-x_i}$$

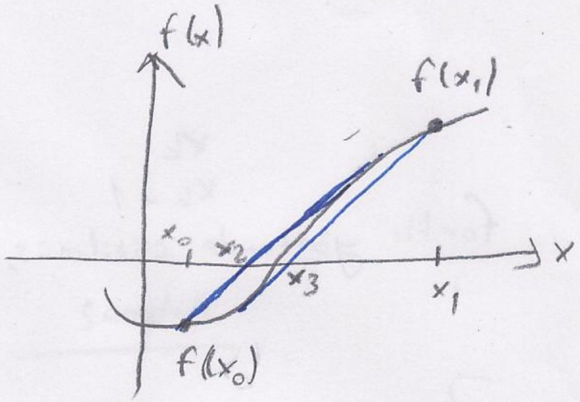
in general form

$$x_{k+1} = x_k - f(x_k)\frac{(x_k-x_{k-1})}{f(x_k)-f(x_{k-1})}$$

Stopping Criteria = $|f_{(x_k)}| < \varepsilon$

$|x_{k+1}-x_k| < \varepsilon$

Geometric Interpretation =



Ex

find the root $f(x) = x^2 - 2$ by using secant method.

$x_0 = 1,5$ and $x_1 = 1$ and $\mathcal{E} = 0,001$

Sol

$$\#1 \quad x_2 = x_1 - f(x_1)\frac{(x_1 - x_0)}{f(x_1) - f(x_0)} = 1 - (-1).\frac{(1-1,5)}{-1-0,25} = 1,4$$

$$\#2 \quad x_3 = x_2 - f(x_2)\frac{(x_2 - x_1)}{f(x_2) - f(x_1)} = 1,4 - \frac{(-0,04)(1.4-1)}{-0,04-(-1)} = 1,4167$$

$\#3 \quad x_4 = 1,4142$

$\# \quad x_5 = 1,4142$

$\left.\right\}$ root = 1,4142

Algorithm of Muller's Method

- Estimate x_0, x_1, x_2
- $f_0 = f(x_0),\ f_1 = f(x_1),\ f_2 = f(x_2)$

$h_1 = x_1 - x_0$

$h_2 = x_0 - x_2$

$\gamma = \dfrac{h_1}{h_2}$

$c = f_0$

$$a = \frac{\gamma f_1 - f_0(1+\gamma) + f_2}{\gamma h_1^2 (1+\gamma)}$$

$$b = \frac{f_1 - f_0 - a h_1^2}{h_1}$$

$$\text{root}(x_r) = x_0 - \frac{2c}{b \mp \sqrt{b^2 - 4ac}}$$

$\left\{ \begin{array}{l} \text{If } b > 0, \text{ use } + \\ \text{If } b < 0, \text{ use } - \end{array} \right.$

x_0, x_1, x_2, x_r

If $x_r > x_0$

then $x_2 = x_0,\ x_0 = x_r,\ x_1 = x_1$ (x_2, x_0, x_1)

Else $x_r < x_0$

then $x_2 = x_2,\ x_0 = x_r,\ x_1 = x_0$ (x_2, x_0, x_1)

Ex

find the root $f(x) = 3x - \sin(x) - e^x$ by using Muller's Method $\underbrace{x_2 = 0, \quad x_0 = 0.5, \quad x_1 = 1}_{\text{given initial values}}$

① $- f_0 = f(x_0) = 0,330704$

$- f_1 = f(x_1) = 1,123189$

$- f_2 = f(x_2) = -1$

② $h_1 = x_1 - x_0 = 1 - 0,5 = 0,5$

$h_2 = x_0 - x_2 = 0,5 - 0 = 0,5$ (sayılarda hata var)

$\gamma = \dfrac{h_1}{h_2} = \dfrac{0,5}{0,5} = 1$

$c = f_0 = 0,330704$

③ $a = \dfrac{(1)(1.123189) - (0,330704)(2) + (-1)}{1(0,5)^2 . 2} = -1,07644$

$b = \dfrac{1,123189 - 0,330704 - (-1,07644).(0,5)^2}{0,5} = 2,12319$

$c = 0,330704$

$- Y_r = 0,5 - \dfrac{2(0,330704)}{2.123189 + \sqrt{(2,1213189)^2 - 4(-1,076444}}$

ilk ilterasyon $= 0,354914$

Next iteration

$x_r < x_0$, so $x_2 = x_0$, $x_0 = x_r$, $x_1 = x_1$

$x_2 = 0$

$x_0 = 0,354914$

$x_1 = 0,5$

\# 2 $f(0) = f(x_0) = -0{,}0138066$

$f_1 = f(x_1) = 0{,}330704$

$f_2 = f(x_2) = -1$

$-h_1 = 0{,}145086$

$h_2 = 0{,}354914$

$\gamma = 2{,}44623$, $c = f(x_0) = -0{,}0138066$

$-a = -0{,}808314$

$b = 2{,}49180$

$$\text{root } (x_r) = 0{,}354914 - \frac{2(-0{,}0138066)}{2.49180 + \sqrt{(2{,}49180)^2 - 4}}$$

$= \underline{0{,}360465}$

for 3. iteration

If $x_r(0{,}360465) > x_0(0{,}354914)$

$x_2 = x_0, \quad x_0 = x_r, \quad x_1 = x_1$

New three points

$x_0 = 0{,}360465$

$x_1 = 0{,}5$

$x_2 = 0{,}354914$

3. kök $\longrightarrow$ 0,3549

Ex

Find the root $f(x) = e^{-x} - x$ by using secant method.

$x_0 = 0$, $x_1 = 1$, $\varepsilon = 0,01$

Sol

#1 $x_2 = x_1 - \dfrac{f(x_1).(x_0 - x_1)}{F(x_0 - f(x_1)} = 1 - \dfrac{(-0,63212)(0-1)}{1-(-0,63212)}$

$x_2 = 0,61270$

$x_3 = 0,61270 - \dfrac{(-0,07081)(1-0,61270)}{-0,63212-(-0,07081)}$

$x_3 = 0,56384$

$x_4 = 0,56384 - \dfrac{(0,00518)(0,61270-0,56384)}{(-0,07081)-(-0,00518)}$

$x_4 = 0,56717$

$x_4 - x_3 = 0,00367 < 0,01$

root $= 0,56384$

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