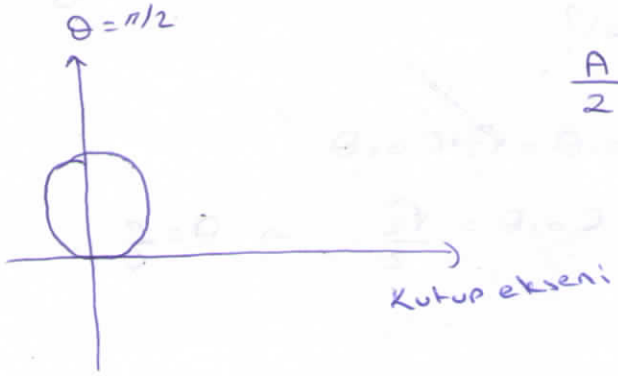
Pınar Albayrak
4. Uygulama

(*) $r = \sqrt{2}\sin\theta$ ile sınırlı bölgenin alanı?



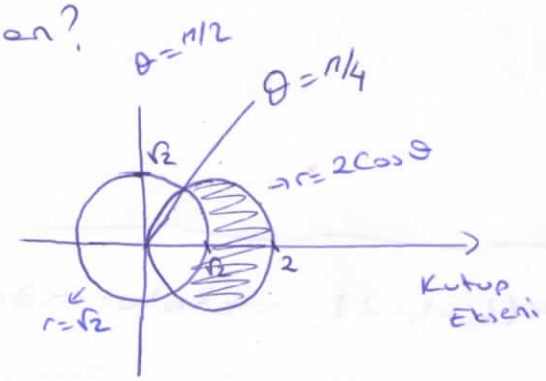
$$\frac{A}{2} = \frac{1}{2}\int_0^{\pi/2} (\sqrt{2}\sin\theta)^2\, d\theta$$

$$= \frac{1}{2}\int_0^{\pi/2} 2\sin^2\theta\, d\theta$$

$$= \int_0^{\pi/2} \frac{1-\cos 2\theta}{2}\, d\theta = \frac{\pi}{4}$$

$$\boxed{A = \frac{\pi}{2}}$$

(*) a) $r = 2\cos\theta$ eğrisinin içinde $r = \sqrt{2}$ nin dışında kalan alan?



$$2\cos\theta = \sqrt{2} \rightarrow \boxed{\theta = \frac{\pi}{4}}$$

$$\frac{A}{2} = \frac{1}{2}\int_0^{\pi/4} (2\cos\theta)^2\, d\theta - \frac{1}{2}\int_0^{\pi/4} (\sqrt{2})^2\, d\theta$$

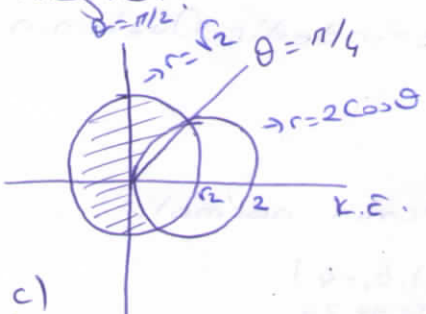
$$= \frac{1}{2}\left[\int_0^{\pi/4} (4\cos^2\theta - 2)\, d\theta\right]$$

$$= \frac{1}{2}\int_0^{\pi/4} 2\cos 2\theta\, d\theta = \frac{\sin 2\theta}{2}\Big|_0^{\pi/4} = \frac{1}{2}$$

$$\boxed{A = 1}$$

b)

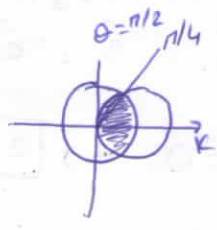
✯ $r = 2\cos\theta$ nın dışında, $r = \sqrt{2}$ nin içinde kalan alanı veren integral:



$$\frac{A}{2} = \frac{1}{2}\int_{\pi/4}^{\pi} (\sqrt{2})^2 d\theta - \frac{1}{2}\int_{\pi/4}^{\pi/2} (2\cos\theta)^2 d\theta$$

c)

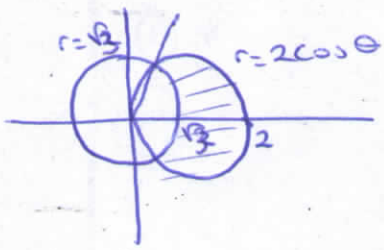
✯ Ortak Alanı veren integral:



$$\frac{A}{2} = \frac{1}{2}\int_0^{\pi/4} (\sqrt{2})^2 d\theta + \frac{1}{2}\int_{\pi/4}^{\pi/2} (2\cos\theta)^2 d\theta$$

(*)

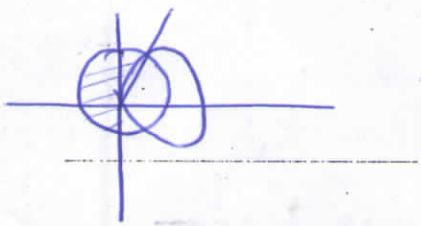
a) $r=2\cos\theta$ içinde, $r=\sqrt{3}$ dışında kalan bölgenin alanını veren integral?



$$\frac{A}{2}=\frac{1}{2}\int_0^{\pi/6}\left((2\cos\theta)^2-3\right)d\theta$$

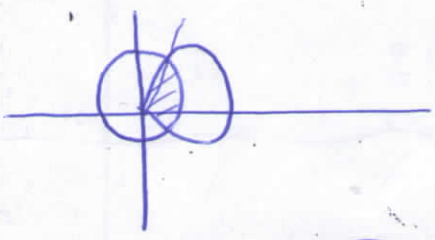
$2\cos\theta=\sqrt{3} \quad \theta=\frac{\pi}{6}$

b) $r=2\cos\theta$ dışında, $r=\sqrt{3}$ içindeki alan?



$$\frac{A}{2}=\frac{1}{2}\int_{\pi/6}^{\pi} 3\,d\theta-\int_{\pi/6}^{\pi/2}(2\cos\theta)^2 d\theta$$

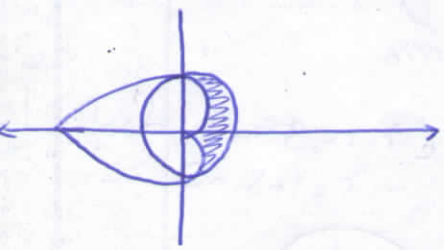
c) $r=2\cos\theta$, $r=\sqrt{3}$ içindeki ortak alan?



$$\frac{A}{2}=\frac{1}{2}\int_0^{\pi/6}3\,d\theta+\frac{1}{2}\int_{\pi/6}^{\pi/2}(2\cos\theta)^2 d\theta$$

(*) $r=1$ içinde

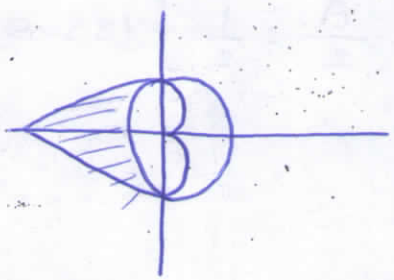
$r=1-\cos\theta$ dışında.



$$\frac{A}{2}=\frac{1}{2}\int_0^{\pi/2}\left(1-(1-\cos\theta)^2\right)d\theta$$

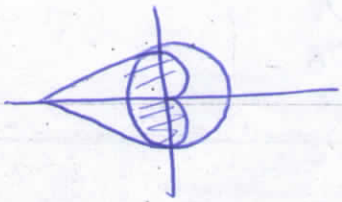
(*) $r=1$ dışında

$r=1-\cos\theta$ içinde



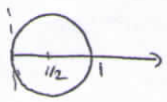
$$\frac{A}{2}=\frac{1}{2}\int_{\pi/2}^{\pi}\left((1-\cos\theta)^2-1^2\right)d\theta$$

(*) $r=1$, $r=1-\cos\theta$ ortak alan?



$$\frac{A}{2}=\frac{1}{2}\int_0^{\pi/2}(1-\cos\theta)^2 d\theta+\frac{1}{2}\int_{\pi/2}^{\pi}1^2\,d\theta$$

(*) $r = \cos\theta$ çemberinin uzunluğu?



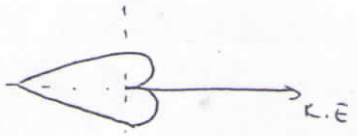
$r = \cos\theta \qquad r' = -\sin\theta$

$r^2 + (r')^2 = \cos^2\theta + \sin^2\theta = 1$

$\sqrt{r^2 + (r')^2} = 1$

$$S = \int_{-\pi/2}^{\pi/2} d\theta = \theta \Big|_{-\pi/2}^{\pi/2} = \frac{\pi}{2} + \frac{\pi}{2} = \pi$$

(*) $r = 1 - \cos\theta$ kardiyoidinin uzunluğu?



$r = 1 - \cos\theta \qquad r' = \sin\theta$

$r^2 + (r')^2 = 1 - 2\cos\theta + \cos^2\theta + \sin^2\theta = 2 - 2\cos\theta$

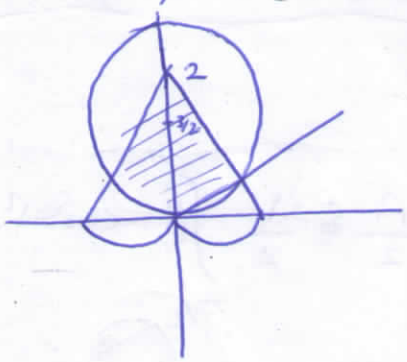
$= 2 - 2\left[1 - 2\sin^2\frac{\theta}{2}\right] = 4\sin^2\frac{\theta}{2}$

$$\sqrt{r^2 + (r')^2} = \sqrt{4\sin^2\frac{\theta}{2}} = \left|2\sin\frac{\theta}{2}\right|$$

$$S = \int_0^{2\pi} \left|2\sin\frac{\theta}{2}\right| d\theta = \int_0^{2\pi} 2\sin\frac{\theta}{2}\, d\theta = -4\cos\frac{\theta}{2}\Big|_0^{2\pi} = 4 + 4 = 8$$

---

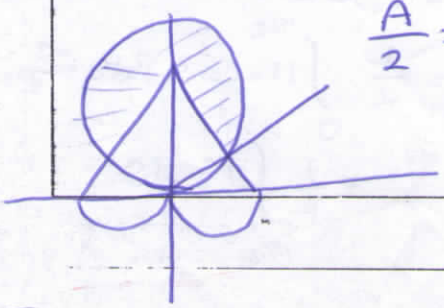
a) $r = 3\sin\theta$, $r = 1 + \sin\theta$ ortak alan?



$3\sin\theta = 1 + \sin\theta \qquad \sin\theta = \frac{1}{2} \qquad \theta = \frac{\pi}{6}$

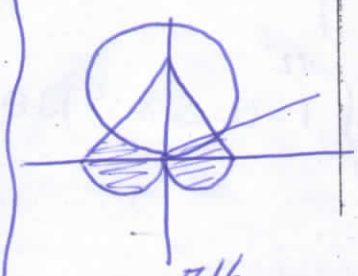
$$\frac{A}{2} = \frac{1}{2}\int_0^{\pi/6} (3\sin\theta)^2 d\theta + \frac{1}{2}\int_{\pi/6}^{\pi/2} (1+\sin\theta)^2 d\theta$$

b) $r = 3\sin\theta$ içi
$r = 1 + \sin\theta$ dışı



$$\frac{A}{2} = \frac{1}{2}\int_{\pi/6}^{\pi/2} (3\sin\theta)^2 - (1+\sin\theta)^2 d\theta$$

c) $r = 3\sin\theta$ dışı
$r = 1 + \sin\theta$ içi



$$\frac{A}{2} = \frac{1}{2}\int_{-\pi/2}^{\pi/6} (1+\sin\theta)^2 d\theta - \frac{1}{2}\int_{0}^{\pi/6} (3\sin\theta)^2$$

YTÜ - Fen-Edebiyat Fakültesi Sınav Soru ve Cevap Kağıdı				NOT TABLOSU				
				1. S	2. S	3.S	4.S	TOPLAM
Adı Soyadı								
Öğrenci Numarası		Grup No						
Bölümü			Sınav Tarihi	06/04/2013				
Dersin Adı	MATEMATİK II		Sınav Süresi	90 dk	Sınav Yeri			
Dersi veren Öğretim Üyesinin Adı Soyadı			İmza					

YÖK nun 2547 sayılı Kanunun ***Öğrenci Disiplin Yönetmeliğinin*** 9. Maddesi olan ***"Sınavlarda kopya yapmak ve yaptırmak veya buna teşebbüs etmek"*** fiili işleyenler **bir veya iki yarıyıl uzaklaştırma** cezası alırlar.

1) $\lim_{x\to\infty}[6x^5\sin(\frac{1}{x})-6x^4+x^2]$ limitini seri açılımlarından faydalanarak çözünüz.

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

$$\sin\frac{1}{x} = \frac{1}{x} - \frac{1}{x^3 3!} + \frac{1}{5!x^5} - \cdots$$

$$\lim_{x\to\infty}\left[6x^5.\sin\frac{1}{x} - 6x^4+x^2\right] = \lim_{x\to\infty}\left[6x^5\left(\frac{1}{x} - \frac{1}{3!x^3} + \frac{1}{5!x^5} - \frac{1}{7!x^7}\cdots\right] - 6x^4 + x^2\right.$$

$$= \lim_{x\to\infty}\left[6x^4 - x^2 + \frac{1}{20} - \frac{6}{7!}\cdot\frac{1}{x^2} - \cdots\right] - 6x^4 + x^2$$

$$= \lim_{x\to\infty}\left[\frac{1}{20} - \underbrace{\frac{6}{7!}\cdot\frac{1}{x^2} + \cdots}_{0}\right] = \boxed{\frac{1}{20}}$$

(*) $r=4$, $\theta=\frac{\pi}{2}$, $r=2\sec\theta$ arasında kalan alanı kutupsal integral ile hesaplayın.

$r=2\sec\theta = \frac{2}{\cos\theta} \Rightarrow r\cos\theta = 2$ $\boxed{x=2}$ doğrusu

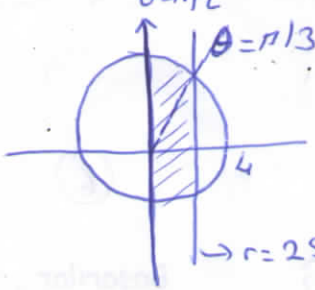
$r\cos\theta = 2 \Rightarrow r = \frac{2}{\cos\theta}$

$r=4 \Rightarrow 4 = \frac{2}{\cos\theta}$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

$r=4 \rightarrow$ 4 yarıçaplı merkezil çember

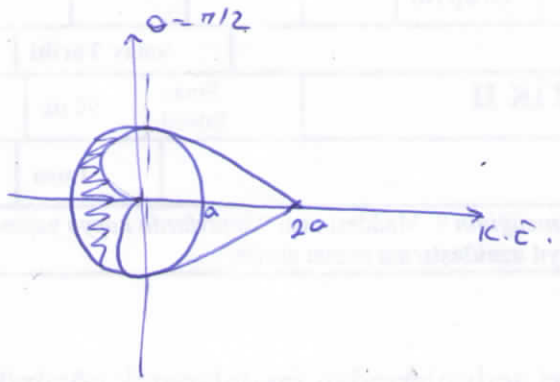


$$\frac{A}{2} = \frac{1}{2}\int_0^{\pi/3}(2\sec\theta)^2 d\theta + \frac{1}{2}\int_{\pi/3}^{\pi/2} 4^2\, d\theta$$

$$A = 4\tan\theta\Big|_0^{\pi/3} + 16\theta\Big|_{\pi/3}^{\pi/2} = 4\sqrt{3} + \frac{8\pi}{3}$$

Başarılar...

2) $a>0$ olmak üzere $r=a(1+\cos\theta)$ kardiyoidinin dışında, $r=a$ çemberinin içinde kalan bölgenin alanını hesaplayınız. (Şekil çiziniz)



$$\frac{A}{2}=\int_{\pi/2}^{\pi} a^2-(a+a\cos\theta)^2\,d\theta=\int_{\pi/2}^{\pi}(-2a^2\cos\theta-a^2\underbrace{\cos^2\theta}_{\frac{1+\cos 2\theta}{2}})\,d\theta$$

$$=-2a^2\sin\theta-a^2\frac{\theta}{2}-\frac{a^2}{4}\sin 2\theta\Big|_{\pi/2}^{\pi}$$

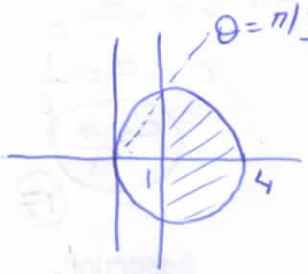
$$=-a^2\frac{\pi}{2}-\left(-2a^2-a^2\frac{\pi}{4}\right)=-\frac{a^2\pi}{2}+2a^2+\frac{a^2\pi}{4}$$

$$=2a^2-\frac{\pi}{4}a^2$$

(*) $r=4\cos\theta$ ve $r\cos\theta\geq 1$ arasında kalan alanı veren integrali yazıp alanı bulunuz.

$r=4\cos\theta \Rightarrow$ Ötelenmiş Çember, $r\cos\theta\geq 1 \Rightarrow \boxed{x\geq 1}$

$r\cos\theta=1 \Rightarrow r=\frac{1}{\cos\theta}$ $\quad r=4\cos\theta \Rightarrow 4\cos\theta=\frac{1}{\cos\theta}$ $\quad \cos^2\theta=\frac{1}{4}$ $\quad \boxed{\theta=\frac{\pi}{3}}$



$$\frac{A}{2}=\frac{1}{2}\int_0^{\pi/3}\underbrace{(4\cos\theta)^2}_{16\left(\frac{1+\cos 2\theta}{2}\right)}d\theta-\frac{1}{2}\int_0^{\pi/3}\underbrace{\left(\frac{1}{\cos\theta}\right)^2}_{\sec^2\theta}d\theta$$

$$A=8\left[\theta+\frac{\sin 2\theta}{2}\Big|_0^{\pi/3}\right]-\tan\theta\Big|_0^{\pi/3}=\frac{8\pi}{3}+\sqrt{3}$$

(6)

Başarılar...

(*) $\begin{cases} x = 8\cos t + 8t\sin t \\ y = 8\sin t - 8t\cos t \\ 0 \le t \le \frac{\pi}{2} \end{cases}$ parametrizasyonu ile verilen eğrinin uzunluğu?

$$S = \int_0^{\pi/2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$\frac{dx}{dt} = -8\sin t + 8\sin t + 8t\cos t \Rightarrow \left(\frac{dx}{dt}\right)^2 = 64t^2\cos^2 t$

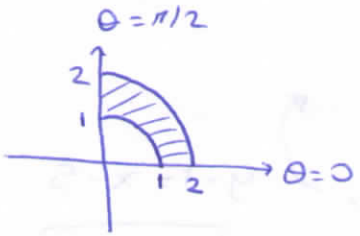
$\frac{dy}{dt} = 8\cos t - 8\cos t + 8t\sin t \Rightarrow \left(\frac{dy}{dt}\right)^2 = 64t^2\sin^2 t$

$$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{64t^2} = 8t$$

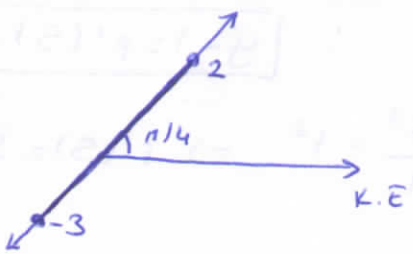
$$S = \int_0^{\pi/2} 8t\, dt = 4t^2 \Big|_0^{\pi/2} = \underline{\underline{\pi^2}}$$

(*) Kutupsal koordinatları aşağıdaki şartları sağlayan noktalar kümesinin grafiğini çiziniz.

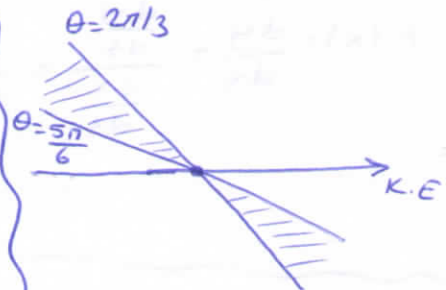
a) $1 \le r \le 2$ ve $0 \le \theta \le \pi/2$



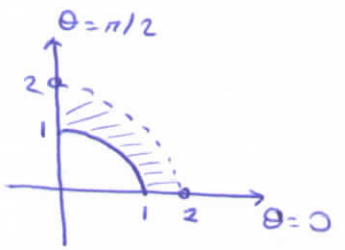
b) $-3 \le r \le 2$ ve $\theta = \frac{\pi}{4}$



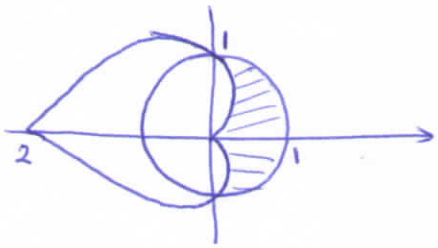
c) $\frac{2\pi}{3} \le \theta \le \frac{5\pi}{6}$



d) $1 \le r < 2$, $0 \le \theta \le \frac{\pi}{2}$



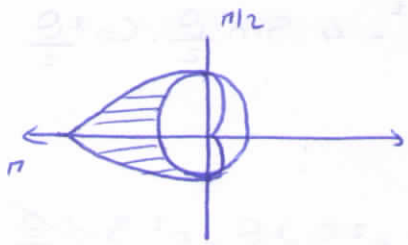
(*) $r=1$ çemberinin içinde, $r=1-\cos\theta$ kardiyoidinin dışında kalan bölgenin alanını veren integral?



$$\frac{A}{2}=\int_0^{\pi/2}\frac{1}{2}.d\theta-\frac{1}{2}\int_0^{\pi/2}(1-\cos\theta)^2\,d\theta$$

$$A=\int_0^{\pi/2}\left(1-(1-\cos\theta)^2\right)d\theta$$

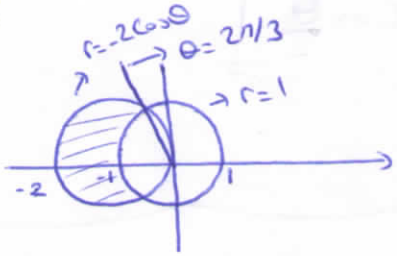
b) çemberin dışı, kardiyoidin içi:



$$\frac{A}{2}=\int_{\pi/2}^{\pi}\frac{1}{2}.(1-\cos\theta)^2\,d\theta-\int_{\pi/2}^{\pi}\frac{1}{2}\,d\theta$$

$$A=\int_{\pi/2}^{\pi}\left((1-\cos\theta)^2-1\right)d\theta$$

(*) $r=-2\cos\theta$ çemberinin içinde, $r=1$ çemberinin dışında kalan alan?



$$-2\cos\theta=1 \Rightarrow \theta=\frac{2\pi}{3}$$

$$\frac{A}{2}=\frac{1}{2}\int_{2\pi/3}^{\pi}(-2\cos\theta)^2\,d\theta-\frac{1}{2}\int_{2\pi/3}^{\pi}1^2\,d\theta$$

$$A=\int_{2\pi/3}^{\pi}\left(4\overbrace{\cos^2\theta}^{\frac{1+\cos2\theta}{2}}-1\right)d\theta=\int_{\frac{2\pi}{3}}^{\pi}(1+2\cos2\theta)\,d\theta$$

$$=\theta+\sin2\theta\Big|_{2\pi/3}^{\pi}=\pi-\frac{2\pi}{3}-\sin\frac{\pi}{3}=\frac{\pi}{3}-\frac{\sqrt{3}}{2}$$

(*) $r = a\sin^2\frac{\theta}{2}$ eğrisinin $0 \le \theta \le \pi$ aralığındaki uzunluğu? $(a>0)$

$$S = \int_0^{\pi} \sqrt{r^2 + (r')^2}\, d\theta$$

$$r^2 = a^2 \sin^4\frac{\theta}{2}$$

$$r' = a \cdot 2 \cdot \sin\frac{\theta}{2} \cdot \cos\frac{\theta}{2} \cdot \frac{1}{2} = a \cdot \sin\frac{\theta}{2} \cdot \cos\frac{\theta}{2}$$

$$(r')^2 = a^2 \cdot \sin^2\frac{\theta}{2} \cdot \cos^2\frac{\theta}{2}$$

$$r^2 + (r')^2 = a^2 \cdot \sin^4\frac{\theta}{2} + a^2 \sin^2\frac{\theta}{2} \cdot \overbrace{\cos^2\frac{\theta}{2}}^{1-\sin^2\frac{\theta}{2}} = a^2 \sin^4\frac{\theta}{2} + a^2 \sin^2\frac{\theta}{2} - a^2 \sin^4\frac{\theta}{2}$$

$$= a^2 \sin^2\frac{\theta}{2}$$

$$\sqrt{r^2 + (r')^2} = \sqrt{a^2 \cdot \sin^2\frac{\theta}{2}} = a \cdot \left|\sin\frac{\theta}{2}\right|$$

$$S = \int_0^{\pi} a \cdot \left|\sin\frac{\theta}{2}\right| d\theta = \int_0^{\pi} a \sin\frac{\theta}{2}\, d\theta = -2a\cos\frac{\theta}{2}\Big|_0^{\pi}$$

$$= \boxed{2a}$$

(*) $r = \cos\theta$, $r = 2\cos\theta$, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{4}$ arasında kalan alan?

$$\frac{A}{2} = \frac{1}{2}\int (2\cos\theta)^2 - (\cos\theta)^2\, d\theta$$

S.3 a) $\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2}$ serisinin toplamını bulunuz.(13p)

$$\frac{2n+1}{n^2(n+1)^2} = \frac{1}{n^2} - \frac{1}{(n+1)^2}$$

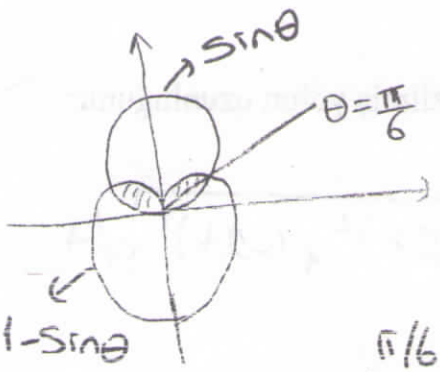
$$S_n = \sum_{k=1}^{n} \frac{2k+1}{k^2(k+1)^2} = \sum_{k=1}^{n} \left(\frac{1}{k^2} - \frac{1}{(k+1)^2} \right)$$

$$= \left(1 - \frac{1}{2^2}\right) + \left(\frac{1}{2^2} - \frac{1}{3^2}\right) + \cdots + \left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right) + \left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right)$$

$$S_n = 1 - \frac{1}{(n+1)^2}$$

$$\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2} = \lim_{n\to\infty} \left(1 - \frac{1}{(n+1)^2}\right) = 1 //$$

b) $\rho = 1 - sin\theta$ kardiyoidi ve $\rho = sin\theta$ çemberinin her ikisinin de içinde kalan bölgenin alanını bulunuz .(12p)



$$1 - Sin\theta = Sin\theta \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\frac{A}{2} = \frac{1}{2}\int_0^{\pi/6} (Sin\theta)^2 d\theta + \frac{1}{2}\int_{\pi/6}^{\pi/2} (1-Sin\theta)^2 d\theta$$

$$A = \int_0^{\pi/6} \frac{1-Cos2\theta}{2} d\theta + \int_{\pi/6}^{\pi/2} \frac{1}{2}(3 - 4Sin\theta - Cos2\theta) d\theta$$

$$A = \left(\frac{\pi}{12} - \frac{\sqrt{3}}{8}\right) + \left(\frac{\pi}{2} - \frac{7\sqrt{3}}{8}\right) = \frac{7\pi}{12} - \sqrt{3} \ br^2$$

Başarılar dilerim

(*) $\left.\begin{matrix} x=4\sin\theta \\ y=2\cos\theta \end{matrix}\right\}$ eğrisinin $\theta=\frac{\pi}{4}$ deki teğeti?

$$m=\frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{-2\sin\theta}{4\cos\theta}\Big|_{\theta=\frac{\pi}{4}}=-\frac{1}{2}$$

$\theta=\frac{\pi}{4} \Rightarrow x_0=2\sqrt{2} \quad y_0=\sqrt{2}$

Teğet : $y-y_0=f'(x_0)(x-x_0) \Rightarrow \boxed{y-\sqrt{2}=-\frac{1}{2}(x-2\sqrt{2})}$

(*) $\left.\begin{matrix} x=2t^2+3 \\ y=t^4 \end{matrix}\right\}$ parametrik denklemi ile verilen eğrinin $t=-1$ deki normal doğrusunun denklemi?

$t=-1 \Rightarrow \begin{matrix} x=5 \\ y=1 \end{matrix} \Rightarrow (5,1)$ den geçen normal : $y-1=m_N(x-5)$

$$m_T=\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{4t^3}{4t}=t^2 \Rightarrow m_T=t^2\Big|_{t=-1}=1 \Rightarrow m_T\cdot m_N=-1$$

$$\Downarrow$$

$$\boxed{m_N=-1}$$

$y-1=-(x-5) \Rightarrow \boxed{y=-x+6}$

(*) $\left.\begin{matrix} x(t)=t^2 \\ y(t)=1-t^2 \end{matrix}\right\}$ eğrisinin $-1\le t\le 0$ aralığındaki uzunluğu?

$$S=\int_{-1}^{0}\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt=\int_{-1}^{0}\sqrt{(2t)^2+(-2t)^2}\,dt=\int_{-1}^{0}2\sqrt{2}\cdot|t|\,dt$$

$$=-2\sqrt{2}\int_{-1}^{0}t\,dt=-2\sqrt{2}\,\frac{t^2}{2}\Big|_{-1}^{0}=\underline{\underline{\sqrt{2}}}$$