| FIZ1002 Physics-2 Midterm-I | | | P1 | P2 | P3 | P4 | TOTAL |
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| Name Surname | | | | | | | |
| Registration No | | | | | | | |
| Department | | | | | | | |
| Group No | Exam Hall | Signature of the Student | | | | | |
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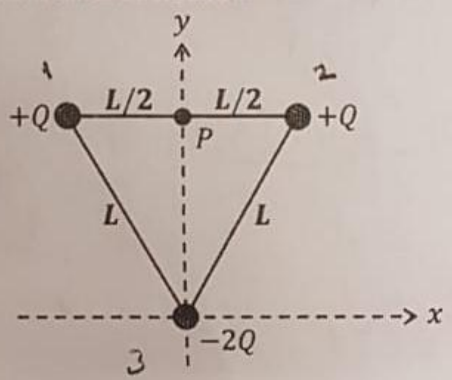
| Lecturer's Name Surname | | | | | | | |

The 9th article of Student Disciplinary Regulations of YÖK Law No.2547 states ***"Cheating or helping to cheat or attempt to cheat in exams"*** de facto perpetrators **takes one or two semesters suspension** penalty. Calculators are not allowed. Do not ask any questions about the problems. There will be no explanations. Use the allocated areas for your answers and write legible

## PROBLEM 1

Three point charges ($+Q, -2Q$, and $+Q$) are placed at the corners of an equilateral triangle of side $L$. The triangle lies in the horizontal $xy$ plane, as shown in the figure. Point $P$ is at the midpoint of the upper side. Express your answers in terms of some or all of the given quantities and appropriate constants as needed.



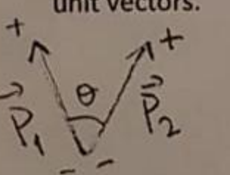

**a)** Find the electric potential $V$ at point $P$.

$$V_P = V_1 + V_2 + V_3$$

$$= k\frac{Q}{\frac{L}{2}} + k\frac{Q}{\frac{L}{2}} - k\frac{2Q}{\frac{\sqrt{3}}{2}L}$$

$$V_P = \frac{4kQ}{L}\left(1 - \frac{1}{\sqrt{3}}\right)$$

**b)** Find the net electrical dipole moment vector $\vec{p}$ of the system of three charges. Express your answer in terms of unit vectors.




$$\vec{P} = \vec{P}_1 + \vec{P}_2 = 2QL\sin 60\,\hat{j}$$

$$\vec{P} = \sqrt{3}\,QL\,\hat{j}$$

$$P_1 = P_2 = QL$$

or

$$\vec{P} = 2Q\frac{\sqrt{3}}{2}L\hat{j} = \sqrt{3}QL\hat{j}$$

2Q, $\frac{\sqrt{3}L}{2}$, −2Q

**c)** Find the total electrostatic potential energy of the system of the three charges.

$$U = k\frac{QQ}{L} - k\frac{2QQ}{L} - k\frac{2QQ}{L}$$

$$U = -\frac{3kQ^2}{L}$$

**d)** Find the torque $\vec{\tau}$ exerted on this dipole placed in an electric field given as $\vec{E} = E_0\hat{\imath}$ . Express your answer in terms of unit vectors.

$$\vec{\tau} = \vec{P}\times\vec{E} = \sqrt{3}QL\hat{j}\times E_0\hat{i}$$

$$\vec{\tau} = -\sqrt{3}LQE_0\hat{k}$$

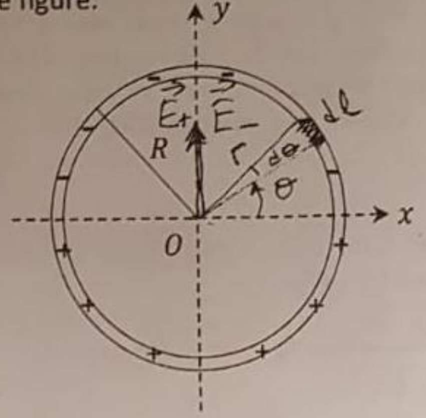
**e)** Find the energy of the dipole placed in an electric field given as $\vec{E} = E_0\hat{\imath}$.

$$U = -\vec{P}\cdot\vec{E}$$

$$= -\sqrt{3}QL\hat{j}\cdot E_0\hat{i}$$

$$U = 0$$

A thin non-conducting rod is bent into a circle of radius $R$. A charge $-Q$ is uniformly distributed along its top half (for $0 < \theta < \pi$) and a charge $+Q$ is uniformly distributed along its bottom half (for $\pi < \theta < 2\pi$), as shown in the figure.




**a)** Find the electric field vector at the origin $O$.

$$\vec{E} = \vec{E}_+ + \vec{E}_-$$

$$\vec{E}_- = E_{-x}\hat{i} + E_{-y}\hat{j}$$

symetric charge distribution about y-axis $\quad E_{-x} = 0$

$$\vec{E}_- = E_- \sin\theta\,\hat{j} = k\int \frac{dq}{r^2}\sin\theta\,\hat{j}$$

$r = R \quad dq = \lambda d\ell \quad d\ell = R\,d\theta$

$$\vec{E}_- = k\int \frac{\lambda d\ell}{r^2}\sin\theta\,\hat{j}$$

$$\vec{E}_- = k\int \frac{\lambda R\,d\theta}{R^2}\sin\theta\,\hat{j}$$

$$\vec{E}_- = k\frac{\lambda}{R}\int_0^{\pi}\sin\theta\,\hat{j} = k\frac{\lambda}{R}\left(-\cos\theta\right)\Big|_0^{\pi}\hat{j}$$

$$\vec{E}_- = k\frac{2\lambda}{R}\hat{j} = \frac{2kQ}{\pi R^2}\hat{j} \qquad \lambda = \frac{Q}{\frac{2\pi R}{2}}$$

$$\vec{E}_+ = \frac{2kQ}{\pi R^2}\hat{j}$$

$$\vec{E} = \frac{4kQ}{\pi R^2}\hat{j}$$

**b)** Find the electrostatic potential $V$ at the origin. Show all your calculations in detail.

$$V = k\int \frac{dq}{r} = k\int \frac{\lambda d\ell}{r}$$

$$V = V_+ + V_-$$

$$V_+ = k\int_0^{\pi} \frac{\lambda \cancel{R}\,d\theta}{\cancel{R}} = k\lambda\pi \qquad \lambda = \frac{Q}{\frac{2\pi R}{2}}$$

$$V_+ = k\frac{Q}{R}$$

$$V_- = -k\frac{Q}{R}$$

$$V = V_+ + V_- = 0$$

**c)** Find the position of a charge $2Q$ to make the total electric field zero at the origin.

$$\vec{E}_T = \vec{E}_{2Q} + \vec{E}_{Ring} = 0$$

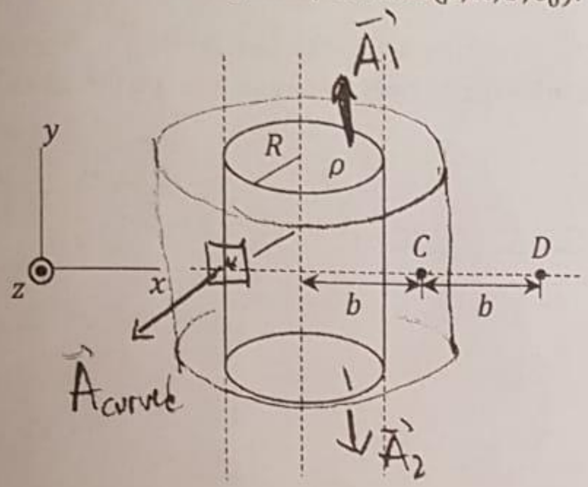
$$k\frac{2Q}{r^2}\hat{r} + 4\frac{kQ}{\pi R^2}\hat{j} = 0$$

$\hat{r} = -\hat{j}$ Direction of $\vec{E}_{2Q}$ $\qquad r^2 = \frac{\pi}{2}R^2$

$$r = \mp\sqrt{\frac{\pi}{2}}R$$

$$r = y = +\sqrt{\frac{\pi}{2}}R$$

An infinitely long non-conducting cylinder has radius $R$ and uniform positive volume charge density $\rho$. The cylinder's axis lies in the $xy$ plane, as shown in the figure. The points $C$ and $D$ are at radial distances $b$ and $2b$, respectively, from the axis of the cylinder. Give your answers in terms of given quantities($\rho, R, b, \varepsilon_0$).




**a) (i)** Using Gauss' law, find the magnitude of the electric field $E(r)$ outside the cylinder at a radial distance $r > R$.

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\oint \vec{E}\cdot d\vec{A} = \int \cancel{\vec{E}\cdot d\vec{A}_1} + \int \cancel{\vec{E}\cdot d\vec{A}_2} + \int \vec{E}\cdot d\vec{A}_{curved} = E\,2\pi r l$$

0 0

$$q_{enc} = \rho V = \rho \pi R^2 l$$

$$E\,2\pi r l = \frac{\rho \pi R^2 l}{\varepsilon_0}$$

$$\boxed{E = \rho R^2/2\varepsilon_0 r}$$

**(ii)** Find the electric field at point $C$. Express your answer in terms of the unit vectors.

$$\vec{E}_c = E(r=b)\,\hat{i}$$

$$= \frac{\rho R^2}{2\varepsilon_0 b}\hat{i}$$

**b)** Using Gauss' law find the electric field $E(r)$ inside the cylinder at a radial distance $r < R$. Indicate the direction of the electric field.

$$\oint \vec{E}\cdot d\vec{A} = \int \vec{E}\cdot d\vec{A}_1 + \int \vec{E}\cdot d\vec{A}_2$$

$$+ \int \vec{E}\cdot d\vec{A}_{curved}$$

$$= E\,2\pi r l$$

$$q_{enc} = \rho V = \rho \pi r^2 l$$

$$E\,2\pi r l = \frac{\rho \pi r^2 l}{\varepsilon_0}$$

$$E = \frac{\rho r}{2\varepsilon_0}$$ (radial direction)

**c)** Find the electric potential difference $V_C - V_D$ between points $C$ and $D$.

$$\Delta V = -\int \vec{E}\cdot d\vec{s}$$

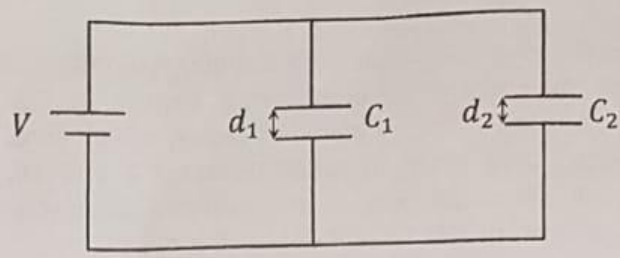
$$= -\int E\,\underbrace{ds\cos\theta}_{-dr}$$

$$= \int_{r_d}^{r_c} \frac{\rho R^2}{2\varepsilon_0 r}\,dr$$

$$= \frac{\rho R^2}{2\varepsilon_0} \ln r \Big|_{2b}^{b}$$

$$\Delta V = \frac{\rho R^2}{2\varepsilon_0} \ln\frac{b}{2b} = \frac{\rho R^2}{2\varepsilon_0}\ln\left(\frac{1}{2}\right)$$

Two parallel-plate capacitors are connected in parallel to a battery, as shown in the figure. Assume that $V = 100\text{V}, C_1 = 1\mu F, C_2 = 2\mu F, d_1 = 2mm,$ and $d_2 = 1mm.$




**a)** Find the charge stored on each capacitor.

$$Q_1 = C_1 V = 10^{-4}\,C$$

$$Q_2 = C_2 V = 2\times10^{-4}\,C$$

**b)** Assume that $d_1$ and $d_2$ are small compared to the dimensions of each plate and find the magnitude of electric field between the plates of each capacitor.

$$V = Ed$$

$$E_1 = \frac{V}{d_1} = 5\times10^{4}\ V/m$$

$$E_2 = \frac{V}{d_2} = 10^{5}\ V/m$$

**c)** Keeping the battery connected, now we insert a dielectric slab with dielectric constant $\kappa = 3$, filling the space between the plates of capacitor $C_1$. Find the charge stored on each capacitor.

$$C_1 = K\,C_{10} = 3\,\mu F$$

$$Q_1 = V C_1 = 3\times10^{-4}\,C$$

$$Q_2 = Q_{20} = 2\times10^{-4}\,C$$

**d)** With the dielectric slab inserted inside $C_1$, find the magnitude of electric field between the plates of each capacitor.

$$E_1 = E_{10} = \frac{V}{d_1} = 5\times10^{4}\ V/m$$

$$E_2 = E_{20} = \frac{V}{d_2} = 10^{5}\ V/m$$

**e)** Find the change in the total energy of the capacitors as the dielectric slab is inserted inside $C_1$.

Final

$$U_1 = K\,U_{10} = K\frac{1}{2}C_{10}V^2$$

$$U_1 = 1.5\times10^{-2}\,J$$

$$U_2 = U_{20} = \frac{1}{2}C_{20}V^2$$

$$U_2 = U_{20} = 10^{-2}\,J$$

$$U_{final} = U_1 + U_2 = 2.5\times10^{-2}$$

initial

$$U_{10} = 0.5\times10^{-2}\,J$$

$$U_{20} = 10^{-2}\,J$$

$$U_{initial} = 1.5\times10^{-2}\,J$$

$$\Delta U = U_f - U_i = 10^{-2}\,J$$