Ex Find $f(2)$ for the same example.

1.0 1.3 1.6 1.9 2.2

To approximate a value when x is close to the end of the tabulated values, say $x = 2.0$, we would again like to make the use of the data points closest to x. This requires using Newton backward divided-difference formula and divided differences in the Table that have a wavy under-line,

x					
1.0					0,0018251
1.3				0,0680685	
1.6			0,0118183		
1.9		-0,5715210			
2.2	0,1103623				

Sol. $f(2,0) = ?$

$P(2,0) = P(x_n + sh) = 0,1103623 - \frac{2}{3} \cdot 0,3(-0,571521)$

$x = x_n + sh$

$2.0 = 2.2 + s\,0.3 \Rightarrow s = \frac{-2}{3}$

$= \frac{-2}{3}\left(\frac{-2}{3}+1\right) \cdot (0,3)^2 \cdot (0,011818)$

$= \frac{-2}{3}\left(\frac{-2}{3}+1\right)\left(\frac{-2}{3}+2\right)(0,06806 \cdot (0,3)^3$

$= \frac{-2}{3}\left(-\frac{2}{3}+1\right)\left(\frac{-2}{3}+2\right)\left(\frac{-2}{3}+3\right) \cdot (0,3)^4 \cdot (0,001825)$

$f(2,0) \cong P(2,0) = 0,2238754$

Norton Interpolating

3- Centered Differences

The Newton forward and backward-differences formulas are not appropriate when x lies near the center of the table. This formula (approximation) is given by stirling's formula.

$$P_n(x) = P_{2m+1}(x) = f[x_0] + \frac{s.h}{2}\left[f[x_{-1}, x_0] + f[x_0, x_1]\right]$$
$$+ s^2h^2 f[x_{-1}, x_1]$$
$$+ \frac{s(s^2-1).h^3}{2} f\left[x_{-2}, x_{-1}, x_0, x_1\right] + f\left[x_{-1}, x_0, x_1, x_2\right]$$
$$+ \ldots s^2(s^2-1)(s^2-4)\ldots(s^2-(m-1)^2)h^{2m}.f\left[x_{-m}, \ldots x_m\right]$$
$$+ \frac{s(s^2-1)\ldots(s^2-m^2)}{2}.h^{2m+1}.f\left[x_{-m}, \ldots x_m\right] + f\left[x_{-m}, \ldots, x_{m+1}\right]$$

If $n=2m+1$ is add, if $n=2m$ is even, we use the same formule. The entries used for this formula are double underlined in Table.

x_i	$F[x_i]$	$f[x_{i-1}, x_i]$	$f[x_{i-2}, x_{i-1}, x_i]$	$f[x_{i-4} \ldots x_i]$	
1.0	0,7651977				
		0,4837057			
1.3	0,6200860		-0,1687339		
		0,5489460		0,0658784	
1.6	0,4554022		-0,0494433		0,0018251
		0,5786120			
1.9	0,2818186			0,0680685	
		0,5715210	-0,0118183		
2.2	0,1103623				

Ex $f(1,5) = ?$ $\begin{pmatrix} 1.0, 1.3, 1,6 \\ 1,9, 2.2 \end{pmatrix}$

Sol. $f(1,5) \cong P(1,5) = P(x_0 + sh)$

$x = x_0 + s.h$ $\quad$ 2, 3, 5, 10, 12, 15

$1,5 = 1,6 + s.(0.3)$

$$= 0,4554022 + \frac{\left(-\frac{1}{3}\right)(0,3)}{2}\left[-0,5489460 + (-0,5786120)\right]$$

$$+ \left(\frac{-1}{3}\right)^2 . (0,3)^2 \left[-0,0494433\right]$$

$$+ \frac{1}{2}\left(\frac{-1}{3}\right)\left[\left(\frac{-1}{3}\right)^2 - 1\right](0.3)^3\left[0,0658784 + 0,0680685\right]$$

$$+ \left(\frac{-1}{3}\right)^2\left[\left(-\frac{1}{3}\right)^2 - 1\right](0,3)^4\left[0,0018251\right]$$

$$= 0,511820$$

Ex

x	0,1	0,2	0,3	0,4	0,5	0,6
$f(x)$	1,172	1,008	0,878	0,782	0,720	0,692

$f(0,15) = ?$ $\quad$ $f(0,36) = ?$ $\quad$ $f(0,55) = ?$

forward $\quad$ Centered $\quad$ Backward

i	x		i	$[x_n]$
0	0,1 $(x_0)[x_0]$	x_{-2}		
1	0,2 (x_1)	x_{-1}		
2	0,3 (x_2)	$\to x_0 = 0,3$		
3	0,4 (x_3)	x_1		
4	0,5 (x_4)	x_2		
5	0,6 $(x_5)[x_n]$	x_3		

$x = x_0 + sh$ $\quad$ $x = x_c + sh$ $\quad$ $x = x_n + sh$

x	f(x)	1.DD	2.DD	3.DD	4.DD	5.DD
0,1	1,172	−0,164	0,034	0	0	0
0,2	1,008	−0,130	0,034	0	0	
0,3	0,878	−0,096	0,034	0		
0,4	0,782	−0,062	0,034			
0,5	0,720	−0,028				
0,6	0,692					

Ex

x	−1	0	0,5	1	2,5	3
f(x)	3	−2	−0,375	3	16,125	19

find f(2) by divided differences.

Sol

forword, centered, backword, the number series of x should be evenly (equally) spaced.

$$\underbrace{1.0 \quad 1.3}_{0.3} \quad \underbrace{1.3 \quad 1.6}_{0.3} \quad \underbrace{1.6 \quad 1.9}_{0.3} \quad \underbrace{1.9 \quad 2.2}_{0.3} \quad 2,5$$

h = 0.3 0.3 0.3 0.3

$$\underbrace{-1 \quad 0}_{1} \quad \underbrace{0 \quad 0,5}_{0,5} \quad \underbrace{0,5 \quad 1}_{0,5} \quad \underbrace{1 \quad \ }_{1,5}$$