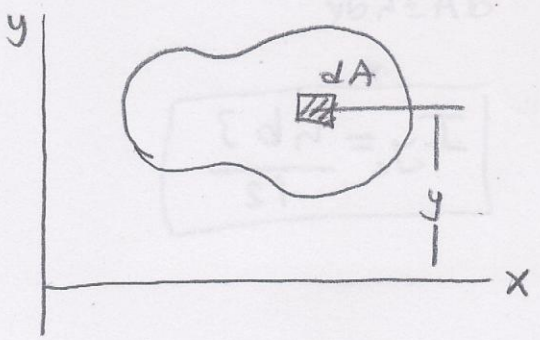
Atalet Momenti



Hatırlatma

Eğilme

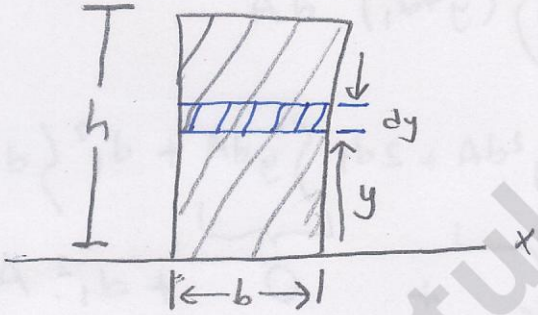
$$\sigma = \frac{M_{max}}{I_x}$$

$$St_x = \int y \cdot dA \qquad St_y = \int x \cdot dA$$

$$I_x = \int y^2 dA \qquad [mm^4]$$

$$I_y = \int x^2 dA$$

Örnek:



$$dA = b\,dy$$

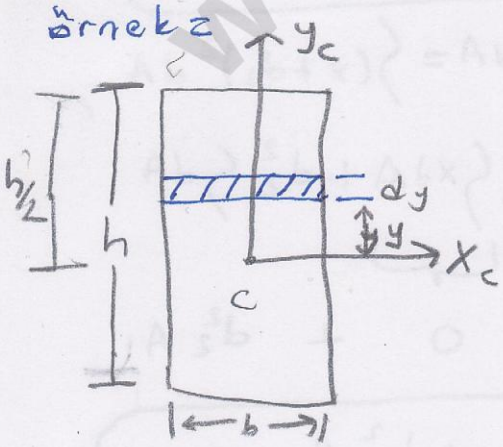
$$I_x = \int y^2 dA$$

$$= b \int_0^h y^2 dy$$

$$= b \cdot \frac{y^3}{3}\Big|_0^h$$

$$= I_x = \frac{b h^3}{3}$$

Örnek:

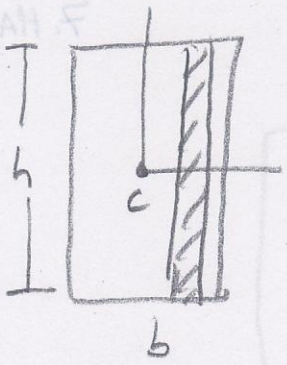


$I_{xc} = ?$ $\quad dA = b.dy$

$$I_{xc} = \int y^2 dA$$

$$= 2b \int_0^{h/2} y^2 dy$$

$$I_{x_c} = \frac{6h^3}{12}$$

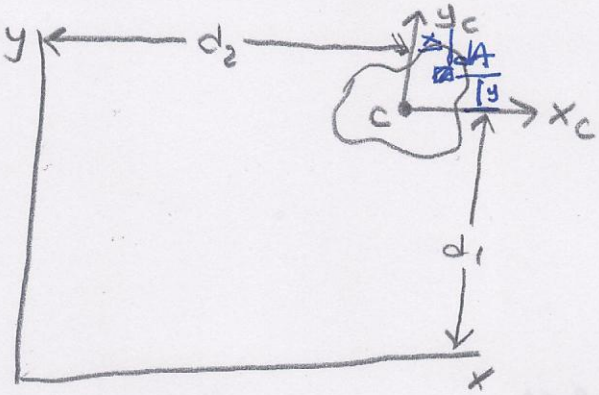


$$I_{y_c} = \int x^2 dA \qquad dA = h\,dx$$

için $\rightarrow$

$$= 2h \int_0^{b/2} x^2 dx \qquad \boxed{I_{y_c} = \frac{hb^3}{12}}$$

Paralel Eksen Teoremi



$I_x = ?$

$$I_x = \int y_1^2 dA \qquad y_1 = y + d_1$$

$$= \int (y + d_1)^2 dA$$

$$= \int y^2 dA + 2d_1 \int y\,dA + d_1^2 \int dA$$

$$I_{x_c} + 0 + d_1^2 \cdot A$$

$$\boxed{I_x = I_{x_c} + d_1^2 \cdot A}$$

$\rightarrow$ x eksenine göre paralel eksen teoremi

$$I_y = \int x_1^2 dA = \int (x + d_2)^2 dA$$

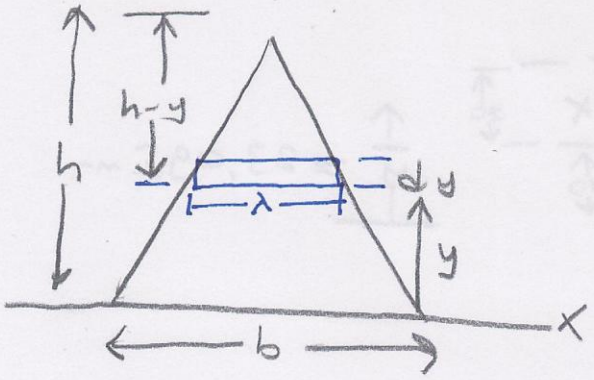
$$I_y = \int x^2 dA + 2d_2 \int x\,dA + d_2^2 \int dA$$

$$I_{y_c} + 0 + d_2^2 A$$

$$\boxed{I_y = I_{y_c} + d_2^2 \cdot A}$$

$\rightarrow$ y eksenine göre paralel eksen teoremi

Üçgen



$I_x = ?$ $\quad dA = \lambda \cdot dy$

$$\frac{\lambda}{b} = \frac{h-y}{h}$$

$$\lambda = \frac{b}{h}(h-y)$$

$$dA = \lambda \cdot dy = \frac{b}{h}(h-y)\,dy$$

$$I_x = \int y^2 dA = \frac{b}{h}\int_0^h y^2 (h-y)\,dy$$

$$\boxed{I_x = \frac{bh^3}{12}}$$

↳ Üçgenin tabanından geçen Atalet Momenti

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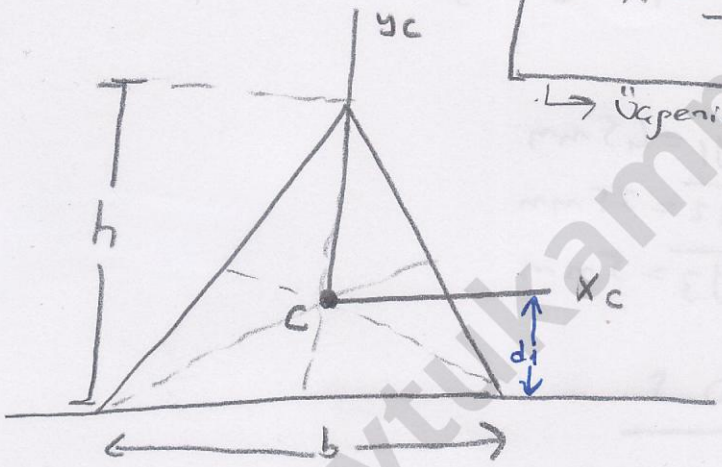


$I_{x_c} = ?$ (Ağırlık merkezinden geçen)

$$d_1 = \frac{h}{3} \qquad A = \frac{b.h}{2}$$

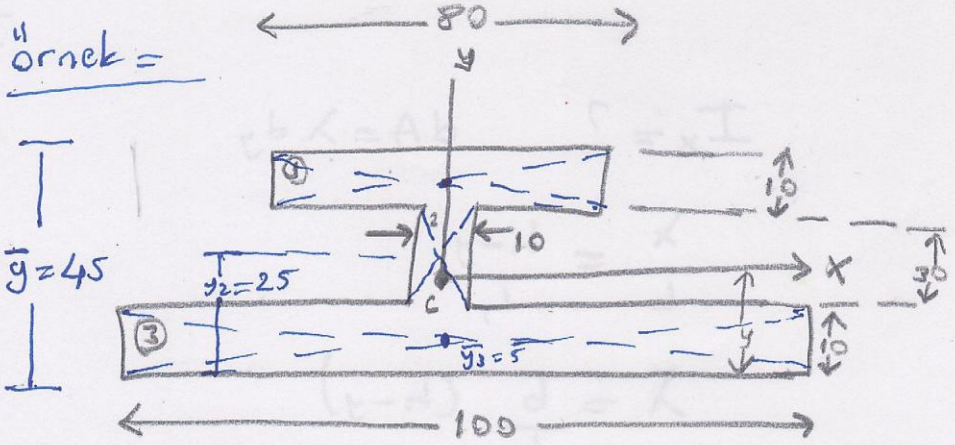
$$I_x = I_{x_c} + d_1^2 A$$

$$= I_x - d_1^2 . A$$

$$I_{x_c} = \frac{b.h^3}{12} - \left(\frac{h}{3}\right)^2 \left(\frac{bh}{2}\right)$$

$$\boxed{I_{x_c} = \frac{bh^3}{36}}$$

4) örnek =



$\bar{y} = 23,095$ mm

$I_x = ?$

Ağırlık merkezini bul.

$$\bar{y} = \frac{\sum_{i=1}^{3} A \cdot \bar{y}_i}{\sum_{i=1}^{3} A_i} = \frac{A_1 \cdot \bar{y}_1 + A_2 \cdot \bar{y}_2 + A_3 \cdot \bar{y}_3}{A_1 + A_2 + A_3}$$

$A_1 = 80 \cdot 10 = 800\ mm^2$ $\quad \bar{y}_1 = 45$ mm

$A_2 = 30 \cdot 10 = 300\ mm^2$ $\quad \bar{y}_2 = 25$ mm

$A_3 = 100 \cdot 10 = 1000\ mm^2$ $\quad \bar{y}_3 = 5$ mm

$$\bar{y} = \frac{800 \cdot 45 + 300 \cdot 25 + 1000 \cdot 5}{800 + 300 + 1000}$$

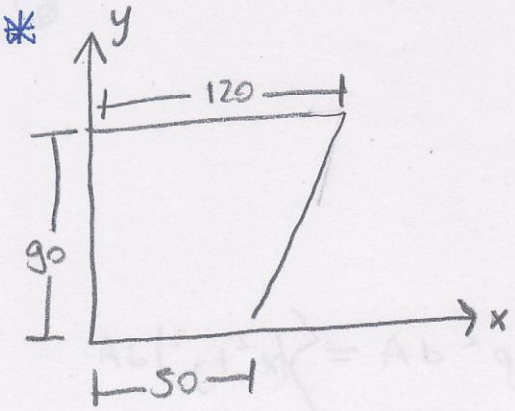
$\bar{y} = 23,095$ mm

$$I_x = I_{x_1} + I_{x_2} + I_{x_3}$$

$$= \left[\frac{b \cdot h_1^3}{12} + (\bar{y}_1 - \bar{y})^2 \cdot A_1\right] + \left[\frac{b_2 h_2^3}{12} + (\bar{y}_2 - \bar{y})^2 \cdot A_2\right] + \left[\frac{b_3 h_3^3}{12} + (\bar{y} - \bar{y}_3)^2 A_3\right]$$

$$= \left[\frac{80 \cdot 10^3}{12} + (45 - 23,095)^2 \cdot 800\right] + \left[\frac{10 \cdot 30^3}{12} + (25 - 23,095)^2 \cdot 300\right] + \left[\frac{100 \cdot 10^3}{12} + (23,095 - 5)^2 \cdot 1000\right]$$

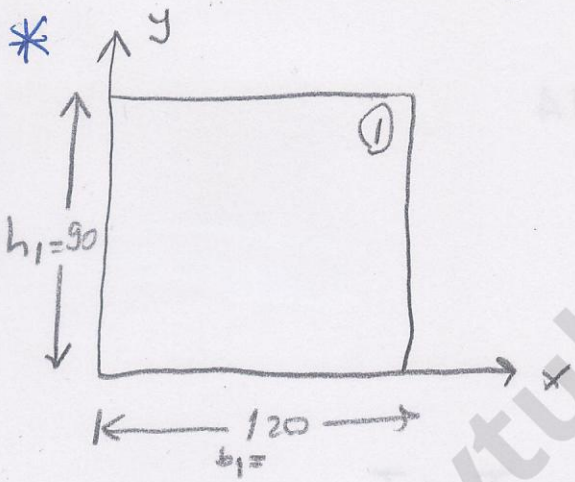
$= \text{-----}\ mm^4$



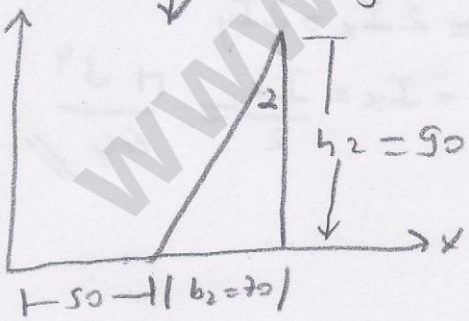
Ölçüler mm cinsindendir.

$I_x = ?$

↓ 1 numaralı dan 2 çıkarılıyor



↓ çıkarılıyor.

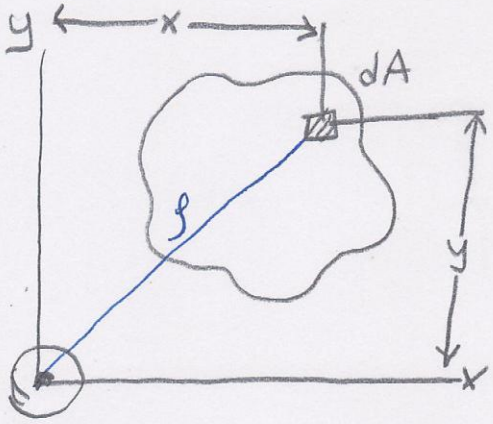


$$I_x = I_{x_1} - I_{x_2}$$

$$= \frac{b_1 h_1^3}{3} - \frac{b_2 h_2^3}{12}$$

$$= \frac{120 \cdot 90^3}{3} - \frac{70 \cdot 90^3}{12} = \text{---} \, mm^4$$

Polar Atalet Momenti



$$I_x = \int y^2 dA$$

$$I_y = \int x^2 dA$$

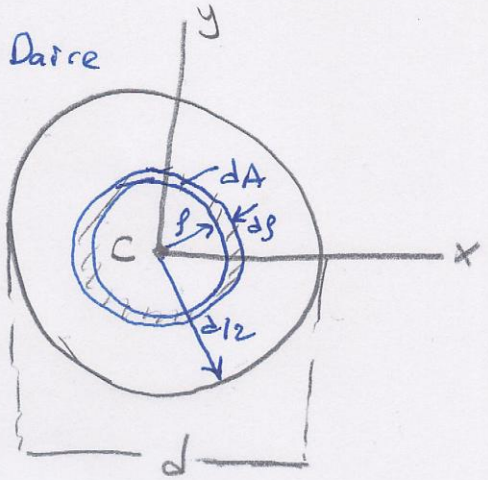
$$I_z = I_{P_o} = \int \rho^2 dA = \int (x^2+y^2) dA$$

$$\rho^2 = x^2+y^2$$

$$= \int x^2 dA + \int y^2 dA$$

$$\boxed{I_{P_o} = I_x + I_y}$$

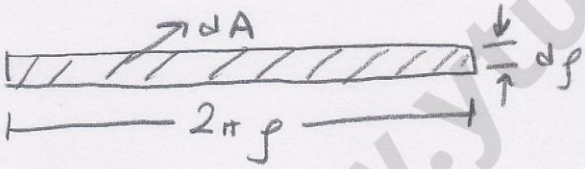
Dolu Daire



$$I_{P_c} = ?$$

$$I_P = \int \rho^2 dA$$

$$dA = 2\pi\rho \cdot d\rho$$



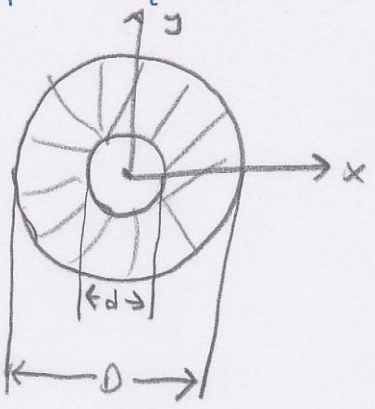
$$I_P = 2\pi \int_0^{d/2} \rho^3 d\rho = \frac{\pi d^4}{32} \;//$$

$$I_{P_c} = I_x + I_y$$

$$I_{P_c} = 2I_x = 2I_y$$

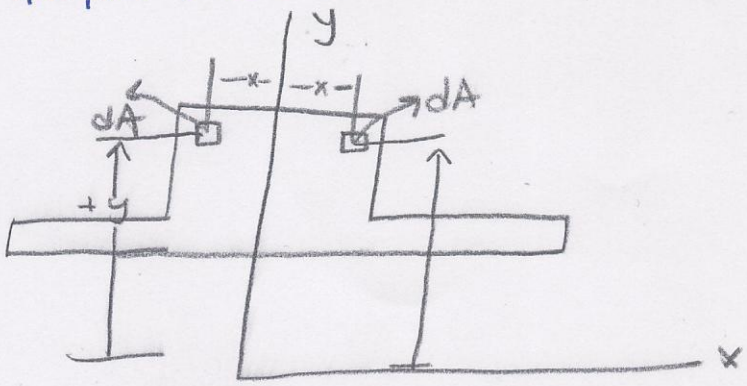
$$I_y = I_x = \frac{I_P}{2} = \frac{\pi d^4}{64} \;//$$

İç: Boş Daire



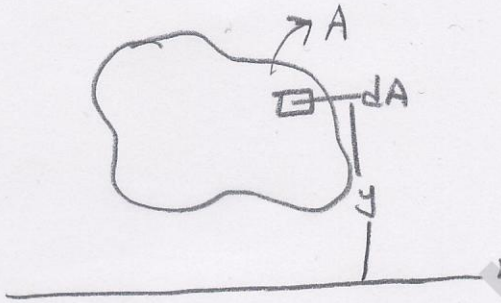
$$I_P = \frac{\pi}{32}(D^4 - d^4)$$

Çarpım Atalet Momentleri (Çıkmaz)

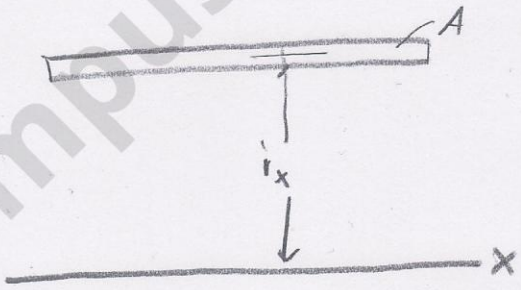


$$I_{xy} = \int xy\,dA$$

Atalet Yarıçapları



$$I_x = \int y^2 dA$$



$$I_x = i_x^2 \cdot A$$

$$i_x = \sqrt{\frac{I_x}{A}}$$