$$\frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy = 0$$

Ödev: $\underbrace{(\tan y - 3x^2)}_{P}dx + \underbrace{x\sec^2 y}_{Q}.dy = 0$

$$\left.\begin{array}{l}\dfrac{\partial P}{\partial y} = \sec^2 y \\ \dfrac{\partial Q}{\partial x} = \sec^2 y\end{array}\right] \text{tam dif.}$$

$$\frac{\partial u}{\partial x} = \tan y - 3x^2 \rightarrow u = \int (\tan y - 3x^2)\,dx + R(y)$$

$$= x\tan y - x^3 + R(y)$$

$$\frac{\partial u}{\partial y} = x\sec^2 y \qquad \frac{\partial u}{\partial y} = x.\sec^2 y + R(y)' = x\sec^2 y$$

$$R(y) = K$$

$$u = x\tan y - x^3 + K = C$$

$$\boxed{x\tan y - x^3 = C_1}$$

6) İntegrasyon Çarpanı

$P(x,y)\,dx + Q(x,y)\,dy = 0$ dif. denk. için; $\dfrac{\partial P}{\partial y} \neq \dfrac{\partial Q}{\partial x}$ ise bu dif. denk. tam dif. denk. değildir. Böyle bir dif. denk. için bazı durumlarda dif. denklemi bir integrasyon çarpanı kullanarak tam dif. haline dönüştürebiliriz.

$\lambda(x,y)$: İntegrasyon çarpanı

Tam dif. olmayan $P(x,y)dx + Q(x,y)dy = 0$ dif. denklemini çarparak bazı durumlarda tam haline dönüştürebiliriz.

$$\underbrace{\lambda(x,y)P(x,y)}_{P_1(x,y)}dx + \underbrace{\lambda(x,y)\,Q(x,y)}_{Q_1(x,y)}dy = 0$$

$$\frac{\partial P_1(x,y)}{\partial y} = \frac{\partial Q_1(x,y)}{\partial x} \Rightarrow \frac{\partial}{\partial y}[\lambda(x,y)P(x,y)] = \frac{\partial}{\partial x}[\lambda(x,y)Q(x,y)]$$

$$\frac{\partial \lambda}{\partial y}.P + \lambda.\frac{\partial P}{\partial y} = \frac{\partial \lambda}{\partial x}.Q + \lambda.\frac{\partial Q}{\partial x}$$

$$\lambda\left[\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right] = \frac{\partial \lambda}{\partial x}.Q - \frac{\partial \lambda}{\partial y}.P$$

Bu dif. denk. kısmi türevli bir dif. denk. olup bu denk. çözmek her zaman mümkün olmayabilir. Bu yüzden bazı özel durumlar için araştırma yapalım.

$\lambda = \lambda(x)$ olma özel hali $\left(\dfrac{\partial \lambda}{\partial x} = \dfrac{d\lambda}{dx};\ \dfrac{\partial \lambda}{\partial y} = 0\right)$

$$\lambda\left[\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right] = \frac{d\lambda}{dx}.Q \rightarrow \frac{d\lambda}{dx}.Q = \lambda\left[\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right]$$

$$\int \frac{d\lambda}{\lambda} = \int \frac{\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}}{Q}.dx$$

$$\boxed{\ln\lambda = \int \frac{\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}}{Q}.dx}$$

x'e bağlı olmalı

$$\ln\lambda = \int \frac{\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}}{P}.dy$$

y'ye

Örn= $\underbrace{(x^2+y^2)}_{P}dx + \underbrace{xy}_{Q}\,dy = 0$

$\left.\begin{array}{l}\dfrac{\partial P}{\partial y} = 2y \\[2ex] \dfrac{\partial Q}{\partial x} = y\end{array}\right] \neq$ Tam dif. değil.

$$\ln\lambda = \int \frac{\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}}{Q}\,dx$$

$$\ln\lambda = \int \frac{2y-y}{xy}\,dx$$

$$\ln\lambda = \int \frac{\cancel{y}}{x\cancel{y}}\,dx = \int \frac{dx}{x} = \ln x = \ln\lambda$$

$$\lambda = x$$

$$x(x^2+y^2)\,dx + x(xy)\,dy = 0$$

$$\underbrace{(x^3+xy^2)}_{P_1}dx + \underbrace{x^2y}_{Q_1}dy = 0$$

$\left.\begin{array}{l}\dfrac{\partial P_1}{\partial y} = 2xy \\[2ex] \dfrac{\partial Q_1}{\partial x} = 2xy\end{array}\right] =$

$\left.\begin{array}{l}\dfrac{\partial u}{\partial x} = x^3+xy^2 \\[2ex] \dfrac{\partial u}{\partial y} = x^2y\end{array}\right\} u(x,y)=c$??

$$\frac{\partial u}{\partial x} = x^3+xy^2 \Rightarrow u = \int (x^3+xy^2)\,dx + R(y)$$

$$u = \frac{x^4}{4} + \frac{x^2y^2}{2} + R(y)$$

$$\frac{\partial u}{\partial y} = x^2y \qquad \frac{\partial u}{\partial y} = 0 + \frac{\cancel{2}x^2y}{\cancel{2}} + \frac{dR(y)}{dy} = x^2y \Rightarrow \frac{dR(y)}{dy} = 0 \Rightarrow R(y) = K$$

$$u = \frac{x^4}{4} + \frac{x^2y^2}{2} + K = C \qquad \boxed{\frac{x^4}{4} + \frac{x^2y^2}{2} = C-K} \quad C_1$$

Örn= $(xy+1).y.dx + (2y-x)\,dy = 0$

$$\underbrace{(xy^2+y)}_{P}dx + \underbrace{(2y-x)}_{Q}dy = 0$$

$\left.\begin{array}{l}\dfrac{\partial P}{\partial y} = 2xy+1 \\[2ex] \dfrac{\partial Q}{\partial x} = -1\end{array}\right] \neq$

$$\ln\lambda = \int \frac{\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}}{Q}\,dx = \int \frac{2xy+1-(-1)}{2y-x}\,dx = \cancel{\int \frac{2xy+2}{2y-x}}\,dx$$

x'e bağlı değil

$$\ln\lambda = \int \frac{\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}}{P}\,dy = \int \frac{(-1)-(2xy+1)}{xy^2+y}\,dy = \int \frac{-2xy-2}{y(xy+1)}\,dy = \int \frac{-2\cancel{(xy+1)}}{y\cancel{(xy+1)}}\,dy = \int \frac{-2}{y}\,dy$$

$$\ln\lambda = -2\ln y = \ln y^{-2} \qquad \lambda = y^{-2} \rightarrow \boxed{\lambda = \frac{1}{y^2}}$$

$$\frac{1}{y^2}(xy^2+y)\,dx + \frac{1}{y^2}(2y-x)\,dy = 0$$

$$\left(x+\frac{1}{y}\right)dx + \left(\frac{2}{y} - \frac{x}{y^2}\right)dy = 0 \qquad \frac{\partial P_1}{\partial y} = -\frac{1}{y^2} \;;\; \frac{\partial Q_1}{\partial x} = \frac{-1}{y^2}$$

GIPTA

$$\left.\begin{aligned}\frac{\partial u}{\partial x} &= x+\frac{1}{y}\\ \frac{\partial u}{\partial y} &= \frac{2}{y}-\frac{x}{y^2}\end{aligned}\right\} \overset{??}{u(x,y)=c}$$

$$\frac{\partial u}{\partial x} = x+\frac{1}{y} \Rightarrow u=\int \left(x+\frac{1}{y}\right)dx + R(y)$$

$$u = \frac{x^2}{2}+\frac{1}{y}x + R(y)$$

$$\frac{\partial u}{\partial y} = 0-\frac{1}{y^2}x+\frac{dR(y)}{dy} = \frac{2}{y}-\frac{x}{y^2}$$

$$\frac{dR(y)}{dy} = \frac{2}{y} \qquad \int dR(y) = \int \frac{2}{y}dy$$

$$R(y) = 2\ln|y|+K$$

$u=\frac{x^2}{2}+\frac{x}{y}+R(y)$ idi

$$u=\frac{x^2}{2}+\frac{x}{y}+2\ln|y|+K \rightarrow \boxed{u=\frac{x^2}{2}+\frac{x}{y}+2\ln|y| = \underbrace{C-K}_{C_1}}$$

Örn: $\underbrace{\left(x-y\sin\frac{y}{x}\right)}_{P}dx+\underbrace{x\sin\frac{y}{x}}_{Q}dy=0$

$$\left.\begin{aligned}\frac{\partial P}{\partial y} &= -1\cdot\sin\frac{y}{x}+-y\cdot\cos\frac{y}{x}\cdot\left(\frac{1}{x}\right)\\ \frac{\partial Q}{\partial x} &= \sin\frac{y}{x}+x\cdot\cos\frac{y}{x}\cdot\left(\frac{-y}{x^2}\right)\end{aligned}\right\} \neq$$

$$\ln\lambda = \int \frac{\frac{\partial P}{\partial y}-\frac{\partial Q}{\partial x}}{Q}dx = \int \frac{\left(-\sin\frac{y}{x}-\cancel{\frac{y}{x}\cos\frac{y}{x}}\right)-\left(\sin\frac{y}{x}-\cancel{\frac{y}{x}\cdot\cos\frac{y}{x}}\right)}{x\sin\frac{y}{x}}\cdot dx = \int\frac{-2\cancel{\sin\frac{y}{x}}}{x\cancel{\sin\frac{y}{x}}}dx$$

$$\ln\lambda = -2\int\frac{dx}{x} = -2\ln x = \ln x^{-2} = \ln\frac{1}{x^2} \qquad \boxed{\lambda = \frac{1}{x^2}}$$

$$\frac{1}{x^2}\left(x-y\sin\frac{y}{x}\right)dx+\frac{1}{x^2}\cdot x\cdot\sin\frac{y}{x}\cdot dy=0$$

$$\underbrace{\left(\frac{1}{x}-\frac{y}{x^2}\sin\frac{y}{x}\right)}_{P_1}dx+\underbrace{\frac{1}{x}\sin\frac{y}{x}}_{Q_1}dy=0$$

$$\left.\begin{aligned}\frac{\partial P_1}{\partial y} &= -\frac{1}{x^2}\cdot\sin\frac{y}{x}+\left(-\frac{y}{x^2}\right)\cdot\cos\frac{y}{x}\cdot\frac{1}{x}\\ \frac{\partial Q_1}{\partial x} &= \frac{-1}{x^2}\cdot\sin\frac{y}{x}+\frac{1}{x}\cdot\cos\frac{y}{x}\cdot\left(\frac{-y}{x^2}\right)\end{aligned}\right\} =$$

$$\left.\begin{aligned}\frac{\partial u}{\partial x} &= \frac{1}{x}-\frac{y}{x^2}\cdot\sin\frac{y}{x}\\ \frac{\partial u}{\partial y} &= \frac{1}{x}\cdot\sin\frac{y}{x}\end{aligned}\right\} \overset{?}{u(x,y)=c}$$

$$\frac{\partial u}{\partial x} = \frac{1}{x}-\frac{y}{x^2}\cdot\sin\frac{y}{x}$$

$$\downarrow$$

$$u=\int\left(\frac{1}{x}-\frac{y}{x^2}\cdot\sin\frac{y}{x}\right)dx + R(y)$$

$$u=\ln x-\cos\frac{y}{x}+R(y)$$

$$\frac{\partial u}{\partial y} = 0+\sin\frac{y}{x}\cdot\frac{1}{x}+\underbrace{\frac{dR(y)}{dy}}_{\to 0} = \frac{1}{x}\cdot\sin\frac{y}{x}$$

$$R(y)=K$$

$$u=\ln x-\cos\frac{y}{x}+K=C$$

$$\boxed{\ln x-\cos\frac{y}{x}=C-K}$$

7) Lineer Dif. Denk.

$$\frac{dy}{dx}+P(x).y=Q(x)$$ şeklindeki dif. denklemlerdir.

<u>1. yol</u>: sabitin değişimi

Öncelikle denklemin ikinci tarafı 0 kabul edilir.

$$\frac{dy}{dx}+P(x).y=0$$

$$\int\frac{dy}{y}=\int-P(x)dx \rightarrow \ln y=-\int P(x)dx+\ln C$$

$$\ln y-\ln C=-\int P(x)dx \rightarrow \ln\frac{y}{c}=-\int P(x)dx \rightarrow \frac{y}{c}=e^{-\int P(x)dx}$$

$$\star\ \boxed{y=c.e^{-\int P(x)dx}} \rightarrow c\overset{??}{=}c(x)$$

c sbt değildir çünkü II. taraf sıfır değil. Q(x), C'yi bulmak için (⋆) ifadesi ve türevleri dif denk'de yerine yazılır.

$$\star\star\quad \frac{dy}{dx}=\frac{dc}{dx}.e^{-\int P(x)dx}+c.(-P(x)).e^{-\int P(x)dx}$$

⋆ ve ⋆⋆ L.D.D.

$$\frac{dc}{dx}.e^{-\int P(x)dx}+\cancel{c.(-P(x)).e^{-\int P(x)dx}}+\cancel{P(x).c.e^{-\int P(x).dx}}=Q(x)$$

$$\frac{dc}{dx}=\frac{Q(x)}{e^{-\int P(x)dx}}=Q(x).e^{\int P(x)dx}$$

$$\int dc=\int Q(x).e^{\int P(x).dx}.dx$$ ⇝ c elde edilir ve ⋆'da yerine yazılır.

örnek $y'=y\cot x+\sin x$

$$\frac{dy}{dx}+(-\cot x).y=\sin x \quad L.D.D$$

$$\frac{dy}{dx}-\cot x.y=0 \longrightarrow \int\frac{dy}{y}=\int\cot x\,dx=\int\frac{\cos x}{\sin x}dx$$

$$\ln|y|=\ln|\sin x|+\ln C$$

$$\ln y=\ln C.\sin x$$

$$\star\ y=C.\sin x \qquad c\overset{?}{=}c(x)$$

$$\star\star\ \frac{dy}{dx}=\frac{dC}{dx}.\sin x+C.\cos x$$

$$\frac{dy}{dx}-\cot x\,y=\sin x \text{ idi}$$

$$\frac{dc}{dx}.\sin x+\cancel{C.\cos x}-\cancel{\cot x.c.\sin x}=\sin x$$

$$\frac{dc}{dx}.\sin x=\sin x \qquad \frac{dc}{dx}=1$$

$$\int dc=\int dx$$

$$c=x+K$$

$y=C.\sin x$ idi

$$y=(x+K)\sin x$$

Örnz $y' + y\sin x = x.e^{\cos x}$ L.D.D

$$\frac{dy}{dx} + y\sin x = 0 \Rightarrow \int \frac{dy}{y} = \int -\sin x dx$$

$$\ln y = \cos x + \ln C \Rightarrow \ln y - \ln C = \cos x \Rightarrow \ln \frac{y}{C} = \cos x$$

$$\frac{y}{C} = e^{\cos x} \Rightarrow \boxed{y = C.e^{\cos x}} \rightsquigarrow C \overset{?}{=} C(x)$$

$\star$ $y = C.e^{\cos x}$

$\star\star$ $\frac{dy}{dx} = \frac{dC}{dx}.e^{\cos x} + C.(-\sin x).e^{\cos x}$ $\star$ ve $\star\star$ L.D.D yerine yaz

$\frac{dy}{dx} + y\sin x = x.e^{\cos x}$ idi. $\frac{dC}{dx}.e^{\cos x} - \cancel{C\sin x e^{\cos x}} + \cancel{C.e^{\cos x}.\sin x} = x.e^{\cos x}$

$$\frac{dC}{dx}.\cancel{e^{\cos x}} = x.\cancel{e^{\cos x}}$$

$$\int dC = \int x dx$$

$$\boxed{C = \frac{x^2}{2} + K}$$

$y = C.e^{\cos x}$ idi

$$\boxed{y = \left(\frac{x^2}{2} + K\right).e^{\cos x}}$$

II. yol: L.D.D için İntegrasyon Çarpanı

$\frac{dy}{dx} + P(x).y = Q(x)$ L.D.D $\mathcal{M}(x)$ int. çarpanı olsun;

$$\mathcal{M}.\frac{dy}{dx} + \mathcal{M}.P.y = \mathcal{M}.Q \rightarrow \frac{dy}{dx}.\mathcal{M} + y.\underbrace{\mathcal{M}.P}_{} = \mathcal{M}.Q$$

$\frac{d\mathcal{M}}{dx}$ olsun → o halde $\mathcal{M} = ?$

$$\frac{d}{dx}(y\mathcal{M}) = \mathcal{M}.Q$$

$$\frac{d\mathcal{M}}{dx} = \mathcal{M}.P$$

$$\int \frac{d\mathcal{M}}{\mathcal{M}} = \int P.dx \rightarrow \ln \mathcal{M} = \int P dx$$

$$\boxed{\# \ \mathcal{M} = e^{\int P(x) dx}}$$

$\frac{d}{dx}(y\mathcal{M}) = \mathcal{M}.Q$ idi

$$\int d(y\mathcal{M}) = \int \mathcal{M}.Q.dx$$

$y\mathcal{M} = \int \mathcal{M}.Q.dx$ Burada # yerine yazalım. $\boxed{y = \frac{1}{\mathcal{M}}\left[\int \mathcal{M}.Q.dx\right]}$

Örnz $y' + y\sin x = x.e^{\cos x}$

$$\left.\begin{array}{l} P(x) = \sin x \\ Q(x) = x.e^{\cos x} \end{array}\right\} \mathcal{M} = e^{\int P(x) dx} = e^{\int \sin x dx} = e^{-\cos x}$$

$$y = \frac{1}{\mathcal{M}}.\left[\int \mathcal{M}.Q dx\right] = \frac{1}{e^{-\cos x}}\left[\int \cancel{e^{-\cos x}}.x.\cancel{e^{\cos x}} dx\right] \rightarrow \boxed{y = e^{\cos x}\left[\frac{x^2}{2} + K\right]}$$