\# $y' = y\cot x + \sin x$

$y' - y\cot x = \sin x$

$\frac{dy}{dx} + y.(-\cot x) = \sin x$

$P(x) = -\cot x$
$Q(x) = \sin x$

$\mu = e^{\int P(x)dx} = e^{\int -\cot x dx} = e^{-\int \cot x dx} = e^{-\int \frac{\cos x}{\sin x} dx}$

$= e^{-\ln \sin x} = \cancel{e}^{\ln (\sin x)^{-1}} = \frac{1}{\sin x}$

$y = \frac{1}{\mu}\left[\int \mu Q dx\right]$

$y = \frac{1}{\frac{1}{\sin x}}\left[\int \frac{1}{\cancel{\sin x}} . \cancel{\sin x} . dx\right] = \boxed{\sin x\,[x + K]}$

8- Bernoulli Dif. Denk.

$\frac{dy}{dx} + P(x).y = Q(x).y^n$

$z = \frac{1}{y^{n-1}} = y^{1-n}$ $\quad \boxed{z = y^{1-n}}$ ☆

$\frac{dz}{dx} = (1-n).y^{\cancel{1-n-1}}.\frac{dy}{dx} \Rightarrow \frac{dz}{dx} = \frac{(1-n)}{y^n}.y' \Rightarrow \boxed{\frac{y'}{y^n} = \frac{1}{1-n}.\frac{dz}{dx}}$ ☆☆

☆ ve ☆☆ Ber. yerine yazılarak L.D.D elde edilir ve çözülür.

$\frac{dy}{dx}^{y'} + P(x).y = Q(x).y^n$

$\frac{y'}{y^n} + P(x).\frac{y}{y^n} = Q(x) \Rightarrow \frac{y'}{y^n} + P(x).\frac{1}{y^{n-1}} = Q(x)$ düzenlenirse LDD elde edilir.

Örn: $y' + y = xy^3$

$\frac{y'}{y^3} + \frac{y}{y^3} = x \longrightarrow \frac{y'}{y^3} + \frac{1}{y^2} = x$

$z = \frac{1}{y^2} = y^{-2}$ $\quad -\frac{1}{2}.\frac{dz}{dx} + z = x \longrightarrow \frac{dz}{dx} - 2z = -2x$ L.D.D

$\frac{dz}{dx} = -2.y^{-2-1}.y' = -2.\frac{y'}{y^3}$

$\left.\begin{array}{l} P(x) = -2 \\ Q(x) = -2x \end{array}\right\} \mu = e^{\int P(x)dx} = e^{\int -2dx} = e^{-2x}$

$\frac{y'}{y^3} = -\frac{1}{2}.\frac{dz}{dx}$

$z = \frac{1}{\mu}.\left(\int \mu.Q\,dx\right) = \frac{1}{e^{-2x}}\left(\underbrace{\int e^{-2x}.-2x.dx}_{I}\right)$

$\frac{dy}{dx} + P(x).y = Q(x)$

$y = \frac{1}{\mu}(\int ---)$

$\frac{dz}{dx} + P(x).z = Q(x)$

$z = \frac{1}{\mu}.(\int ---)$

$\int -2.e^{-2x}.x\,dx$ $\quad = e^{2x}\left(x.e^{-2x} - \int e^{-2x}.dx\right)$

$u = x$ $\quad dv = -2e^{-2x}dx$
$du = dx$ $\quad v = e^{-2x}$

$z = e^{2x}\left(x.e^{-2x} - \frac{e^{-2x}}{-2} + K\right)$

$z = \frac{1}{y^2}$ idi $\quad y = \frac{1}{\sqrt{z}}$

$y = \frac{1}{\sqrt{x + \frac{1}{2} + Ke^{2x}}}$

Örnz $y' + y\tan x = y^3 \sec^3 x$

$$\frac{y'}{y^3} + \frac{y\tan x}{y^3} = \sec^3 x$$

$$z = \frac{1}{y^2} = y^{-2}$$

$$\frac{dz}{dx} = -2y^{-3} \cdot y' \Rightarrow \frac{y'}{y^3} = -\frac{1}{2} \cdot \frac{dz}{dx}$$

$$-\frac{1}{2} \cdot \frac{dz}{dx} + z\tan x = \sec^3 x$$

$$\frac{dz}{dx} - 2z\tan x = -2\sec^3 x \quad \text{L.D.D.}$$

$$\left.\begin{aligned} P(x) &= -2\tan x \\ Q(x) &= -2\sec^3 x \end{aligned}\right\} \mu = e^{\int P(x)dx} = e^{\int -2\tan x\, dx} = e^{2\int -\frac{\sin x}{\cos x} dx} = e^{2\ln(\cos x)}$$

$$\mu = e^{\ln(\cos x)^2} = (\cos x)^2$$

$$z = \frac{1}{\mu}\left(\int \mu . Q . dx\right) = \frac{1}{\cos^2 x}\left[\int \cos^2 x(-2\sec^3 x) dx\right]$$

$$z = \frac{1}{\cos^2 x}\left[\int \cancel{\cos^2 x} . -2 . \frac{1}{\cos^{\cancel{3}} x} dx\right] = \frac{1}{\cos^2 x}\left[-2\int \sec x\, dx\right]$$

$$z = \frac{-2}{\cos^2 x}\left[\ln(\sec x + \tan x) + K\right]$$

$$z = \frac{1}{y^2} \text{ idi } \quad y = \frac{1}{\sqrt{z}} \quad // \quad y = \frac{1}{\sqrt{\frac{-2}{\cos^2 x}\left[\ln(\sec x + \tan x) + K\right]}} \quad //$$

Örnz $y' - \frac{y}{3x} = y^4 \ln x$

$$\frac{y'}{y^4} - \frac{y}{3x . y^4} = \ln x$$

$$\frac{y'}{y^4} - \frac{1}{3x . y^3} = \ln x$$

$$z = \frac{1}{y^3} = y^{-3}$$

$$\frac{dz}{dx} = -3 . y^{-4} . y' \rightarrow \frac{y'}{y^4} = -\frac{1}{3}\frac{dz}{dx}$$

$$-\frac{1}{3} . \frac{dz}{dx} - \frac{z}{3x} = \ln x$$

$$\frac{dz}{dx} + \frac{z}{x} = -3\ln x \quad \text{L.D.D}$$

$$\left.\begin{aligned} P(x) &= \frac{1}{x} \\ Q(x) &= -3\ln x \end{aligned}\right\} \mu = e^{\int P(x)dx} = e^{\int \frac{dx}{x}} = \cancel{e}^{\ln x} = x$$

$$z = \frac{1}{\mu} . \left[\int \mu . Q . dx\right] = \frac{1}{x} . \left[\int x . -3\ln x . dx\right]$$

$$z = \frac{-3}{x} . \left[\int x . \ln x . dx\right]$$

$$u = \ln x \qquad dv = x\,dx$$

$$du = \frac{1}{x} dx \qquad v = \frac{x^2}{2}$$

$$\int x\ln x = \frac{x^2}{2} . \ln x - \int \frac{1}{x} . \frac{x^2}{2} dx$$

$$\int x\ln x = \frac{x^2}{2}\ln x - \frac{1}{4}x^2 + K$$

$$z = \frac{-3}{x}\left[\frac{x^2}{2}\ln x - \frac{1}{4}x^2 + K\right]$$

$$z = \frac{1}{y^3} \text{ idi } \rightarrow y = \frac{1}{\sqrt[3]{z}}$$

$$\boxed{y = \frac{1}{\sqrt[3]{-\frac{3}{x}\left[\frac{x^2}{2}\ln x - \frac{1}{4}x^2 + K\right]}}}$$

9) Riccati Dif. Denk.

$y' + P(x)y^2 + Q(x)y + R(x) = 0$ Bu dif. denk. en az bir y_1 özel çözümü bilinse uygun bir dönüşümle LDD veya Bernoulli'ye dönüştürülerek çözülebilir.

$y = y_1 + z \longrightarrow$ Bernoulli

$y = y_1 + \frac{1}{z} \longrightarrow$ L.D.D

Örn: $y' + y^2 - 1 = 0$ dif. denk. özel bir çözümü $y_1 = 1$ olduğuna göre bu özel çözümden faydalanarak genel çözümü bulunuz.

$y = y_1 + \frac{1}{z}$

$y = 1 + \frac{1}{z}$

$\frac{dy}{dx} = 0 - \frac{1}{z^2} \cdot \frac{dz}{dx}$

$y' + y^2 - 1 = 0$

$-\frac{1}{z^2} \cdot \frac{dz}{dx} + \left(1 + \frac{1}{z}\right)^2 - 1 = 0$

$-\frac{1}{z^2} \cdot \frac{dz}{dx} + \cancel{1} + \frac{2}{z} + \frac{1}{z^2} - \cancel{1} = 0$

$\frac{dz}{dx} - \frac{2z^2}{z} - \frac{\cancel{z^2}}{\cancel{z^2}} = 0$

$\frac{dz_{\shortparallel}}{dx} - 2z_{\shortparallel} = 1 \quad LDD \rightarrow P(x) = -2, \; Q(x) = 1$

$M = e^{\int P(x)dx} = e^{\int -2dx} = e^{-2x}$

$z = \frac{1}{M}\left[\int M.Q.dx\right] = \frac{1}{e^{-2x}}\left[\int e^{-2x}.1.dx\right] = e^{2x}\left[\frac{e^{-2x}}{-2} + c\right] = -\frac{1}{2} + c.e^{2x} = z$

$y = 1 + \frac{1}{z}$ idi $\Rightarrow \boxed{y = 1 + \frac{1}{-\frac{1}{2} + c.e^{2x}}}$

Örn: $y' - \frac{y}{x} + 9x^2 = y^2$ dif. denk. bir özel çözümü $y_1 = 3x$ ise genel çöz.?

$y = 3x + 1/z$

$y' = 3 - 1/z^2 . dz/dx$

$y' - \frac{y}{x} + 9x^2 = y^2$

$\left(3 - \frac{1}{z^2} . \frac{dz}{dx}\right) - \frac{1}{x}\left(3x + \frac{1}{z}\right) + 9x^2 = \left(3x + \frac{1}{z}\right)^2$

$\cancel{3} - \frac{1}{z^2} . \frac{dz}{dx} - \cancel{3} - \frac{1}{zx} + \cancel{9x^2} = \cancel{9x^2} + \frac{6x}{z} + \frac{1}{z^2}$

$\frac{dz}{dx} + \frac{z^2}{zx} = -\frac{6xz^2}{z} - \frac{z^2}{z^2}$

$\frac{dz}{dx} + \left(\frac{1}{x} + 6x\right)z = -1 \rightarrow P(x) = \frac{1}{x} + 6x, \; Q(x) = -1$

$M = e^{\int P(x)dx} = e^{\int\left(\frac{1}{x} + 6x\right)dx} = e^{\ln x + \frac{6x^2}{2}} = e^{\ln x} . e^{3x^2} = x.e^{3x^2}$

$z = \frac{1}{M}\left[\int M.Q.dx\right] = \frac{1}{x.e^{3x^2}}\left[\int xe^{3x^2}(-1)dx\right]$

$= \frac{-1}{6x.e^{3x^2}}\left[\int 6x.e^{3x^2}.dx\right]$

$z = \frac{-1}{6x.e^{3x^2}}\left[e^{3x^2} + c\right]$

$y = 3x + \frac{1}{z}$ idi

$$\boxed{y = 3x + \frac{1}{\left[\frac{-1}{6x} - \frac{c}{6x.e^{3x^2}}\right]}}$$

10) Clairaunt Dif. Denk.

$$y = xy' + \varphi(y')$$

$$y' = p$$

$$y = xp + \varphi(p)$$

$$y' = 1.p + x.\frac{dp}{dx} + \varphi'(p).\frac{dp}{dx}$$

$$\cancel{p} = \cancel{p} + \frac{dp}{dx}\left(x + \varphi'(p)\right)$$

$$\frac{dp}{dx}\left(x + \varphi'(p)\right) = 0$$

1- $\frac{dp}{dx} = 0 \Rightarrow \boxed{p = c = sbt}$ genel çözüm

2- $x + \varphi'(p) = 0$

$$x = -\varphi'(p)$$

$$y = \underline{\underline{x}}p + \varphi(p)$$

$$\left.\begin{aligned} x &= -\varphi'(p) \\ y &= -\varphi'(p).p + \varphi(p) \end{aligned}\right\}$$ Tekil çözümün parametrik denklemi

Eğer bunlar arasında p yok edilir ise tekil çözümün kartezyen denk. bulunur.

Örnek $y = xy' + y'^2$

$y' = p$

"$y = xp + p^2$"

x'e göre

$$y' = 1.p + x.\frac{dp}{dx} + 2p.\frac{dp}{dx}$$

$$\cancel{p} = \cancel{p} + \frac{dp}{dx}(x + 2p)$$

$$0 = \frac{dp}{dx}(x + 2p)$$

1-) $\frac{dp}{dx} = 0 \quad p = c = sbt \Rightarrow$ G.Ç.

2-) $x + 2p = 0 \rightarrow x = -2p$

$$y = \underline{\underline{x}}p + p^2$$

$$x = -2p$$

$$y = -2p^2 + p^2 = -p^2$$

$$\left.\begin{aligned} x &= -2p \\ y &= -p^2 \end{aligned}\right\}$$ Tekil çöz. parametrik denk.

$p = -x/2 \quad y = -p^2$ idi

$$\boxed{y = -\left(-\frac{x}{2}\right)^2}$$ Tekil çöz. kartezyen denk.

Örnek $y = xy' + y' - y'^2$

$y' = p$

"$y = xp + p - p^2$"

$$y' = p + x.\frac{dp}{dx} + \frac{dp}{dx} - 2p.\frac{dp}{dx}$$

$$\cancel{p} = \cancel{p} + \frac{dp}{dx}(x + 1 - 2p)$$

$$\frac{dp}{dx}(x + 1 - 2p) = 0$$

1-) $\frac{dp}{dx} = 0 \quad p = c = sbt$ genel ç.

2-) $x + 1 - 2p = 0$

$$x = 2p - 1$$

$$y = \underline{\underline{x}}p + p - p^2$$

$$x = 2p - 1$$

$$y = (2p - \cancel{1})p + \cancel{p} - p^2 = p^2$$

$$\left.\begin{aligned} x &= 2p - 1 \\ y &= p^2 \end{aligned}\right\}$$ Tekil çöz. parametrik denk.

$$p = \frac{x+1}{2}$$

$$\boxed{y = \left(\frac{x+1}{2}\right)^2}$$ Tekil çöz. kart. denk.

II-) Lagrange Dif. Denk.

$$y = x \cdot f(y') + \varphi(y')$$

$$y' = p$$

$$y = x f(p) + \varphi(p)$$

$$y' = 1 \cdot f(p) + x f'(p) \cdot \frac{dp}{dx} + \varphi'(p) \frac{dp}{dx}$$

$p \rightarrow y'$

$$p - f(p) = \frac{dp}{dx} \left(x \cdot f'(p) + \varphi'(p) \right)$$

$$\frac{dx}{dp} = \frac{x \cdot f'(p) + \varphi'(p)}{p - f(p)} \Rightarrow \frac{dx}{dp} - \frac{f'(p)}{p - f(p)} \cdot x$$

$$= \text{"}\frac{\varphi'(p)}{p - f(p)}\text{"} \quad \text{L.D.D}$$

NOT: $\frac{dy}{dx} + P(x) \cdot y = Q(x)$

$\mu(x) = \ldots$
$y =$

$$\frac{dp}{dx} + P(x) \cdot p = Q(x)$$

$\mu(x) =$
$p =$

$$\frac{dx}{dp} + P(p) \cdot x = Q(p)$$

$$\mu(p) = e^{\int P(p) dp}$$

$$x = \frac{1}{\mu} \left[\int \mu \cdot Q \cdot dp \right]$$

Örn: $y = x(1 + y') + y'^2$

$$y' = p$$

$$\text{"} y = x(1 + p) + p^2 \text{"}$$

$$y' = 1 \cdot (1 + p) + x \cdot \left(0 + \frac{dp}{dx} \right) + 2p \cdot \frac{dp}{dx}$$

$$\cancel{p} = 1 + \cancel{p} + \frac{dp}{dx} (x + 2p)$$

$$-1 = \frac{dp}{dx} (x + 2p)$$

$$\frac{dx}{dp} = -x - 2p$$

$$\frac{dx}{dp} + x = -2p \quad \text{LDD} \quad P(p) = 1, \quad Q(p) = -2p$$

$$\mu(p) = e^{\int P(p) dp} = e^{\int 1 dp} = e^{p}$$

$$x = \frac{1}{\mu} \left[\int \mu \cdot Q \cdot dp \right] = \frac{1}{e^p} \left[\int e^p \cdot (-2p) \, dp \right]$$

$$x = \frac{-2}{e^p} \left[\int p \cdot e^p \cdot dp \right]$$

$$x = \frac{-2}{e^p} \left[p \cdot e^p - e^p + c \right]$$

$$y = x(1 + p) + p^2$$

Örn: $y = 2xy' - y'^2 - 1$

$$y' = p$$

$$y = 2xp - p^2 - 1$$

$$y' = 2p + 2x \cdot \frac{dp}{dx} - 2p \cdot \frac{dp}{dx}$$

$$p = 2p + \frac{dp}{dx} (2x - 2p)$$

$$-p = \frac{dp}{dx} (2x - 2p)$$

$$\frac{dx}{dp} = \frac{2p - 2x}{\ldots}$$

$$\frac{dx}{dp} + \frac{2x}{p} = 2 \quad \text{LDD} \quad P(p) = \frac{2}{p}, \quad Q(p) = 2$$

$$\mu(p) = e^{\int P(p) dp} = e^{\int \frac{2}{p} dp} = e^{2 \ln p} = e^{\ln p^2} = p^2$$

$$x = \frac{1}{\mu} \left[\int Q \cdot \mu \cdot dp \right] \qquad x = \frac{1}{p^2} \left[\int 2 \cdot p^2 dp \right]$$

$$x = \frac{1}{p^2} \left[\frac{2p^3}{3} + c \right]$$

$$y = 2xp - p^2 - 1$$