Uygulama

1- $y=-x(y')^2+\ln y'$

$y'=p \rightarrow y=-xp^2+\ln p$

$y'=-p^2+-x.2p.\frac{dp}{dx}+\frac{1}{p}.\frac{dp}{dx}$

$p^2+p=\frac{dp}{dx}\left(-2px+\frac{1}{p}\right)$ "Lagrange"

$\frac{dx}{dp}=\frac{-2px+\frac{1}{p}}{p^2+p}=\frac{1-2p^2x}{p^3+p^2}=\frac{1}{p^3+p^2}-\frac{2p^2x}{p^3+p^2}$

$\frac{dx}{dp}+\frac{2p^2}{p^3+p^2}.x=\frac{1}{p^3+p^2}$ LDD

$P(p)=\frac{2\cancel{p^2}}{\cancel{p^3+p^2}}$ $p+1$ $\quad Q(p)=\frac{1}{p^3+p^2}$

$\mu=e^{\int P(p)dp}=e^{\int\frac{2}{p+1}.dp}=e^{\ln(p+1)^2}=(p+1)^2$

$x=\frac{1}{\mu}.\left[\int \mu.Q.dp\right]=\frac{1}{(p+1)^2}.\left[\int (p+1)^{\cancel{2}}.\frac{1}{p^2\cancel{(p+1)}}.dp\right]=\frac{1}{(p+1)^2}\left[\int\left(\frac{p}{p^2}+\frac{1}{p^2}\right)dp\right]$

$=\frac{1}{(p+1)^2}\left[\ln p-\frac{1}{p}+c\right]$ $\quad y=-xp^2+\ln p$ genel çözümün parametrik denklemi.

2- $y'+y^2=\frac{2}{x^2}$ dif. denk.nin $y_1=\frac{A}{x}$ bir özel çöz. old. göre $A=?$ Bulunan A değerlerinden en büyüğünü kullanarak dif. denk. $y=y_1+u(x)$ dön. ile çözünüz.

$\left.\begin{array}{l} y=\frac{A}{x} \\ y'=-\frac{A}{x^2}\end{array}\right\}$ $\frac{-A}{x^2}+\frac{A^2}{x^2}=\frac{2}{x^2}$

$A^2-A-2=0$

$(A-2)(A+1)=0$

$A=2,\ A=-1$

$y_1=\frac{2}{x}$, $y=y_1+u(x)$ Bernoulli

$y=\frac{2}{x}+u \Rightarrow y'=-\frac{2}{x^2}+\frac{du}{dx}$

$\cancel{\frac{-2}{x^2}}+\frac{du}{dx}+\cancel{\frac{4}{x^2}}+\frac{4u}{x}+u^2=\cancel{\frac{2}{x^2}}$

$\frac{du}{dx}+\frac{4}{x}.u=-u^2 \Rightarrow P(x)=\frac{4}{x}\quad Q(x)=-1$

$z=u^{1-2}=\frac{1}{u} \Rightarrow \frac{dz}{dx}=-\frac{1}{u^2}.\frac{du}{dx} \Rightarrow \frac{-1}{u^2}.\frac{du}{dx}-\frac{4}{x}.\frac{1}{u}=1 \Rightarrow \frac{dz}{dx}-\frac{4}{x}.z=1$ LDD

$P(x)=-\frac{4}{x}\quad Q(x)=1$

$\mu=e^{\int-\frac{4}{x}dx}=x^{-4}\quad z=\frac{1}{x^{-4}}\left[\int x^{-4}.1.dx\right]=x^4.\left[\int x^{-4}dx\right]$

$=x^4\left[-\frac{1}{3x^3}+c\right]=\frac{-x}{3}+x^4c\quad z=\frac{1}{u}$ idi $\quad u=\frac{1}{-\frac{x}{3}+x^4c}$

$y=\frac{2}{x}+u \rightarrow$ g.ç

3- $2y-t^3\cos t-ty'=0$ $(t>0)$ dif. denk g.ç=?

$\frac{2y}{t}-t^2\cos t-y'=0$

$y'-\frac{2}{t}y=-t^2\cos t$ LDD

$P(t)=\frac{-2}{t}\quad Q(t)=-t^2\cos t$

$\mu=e^{\int-\frac{2}{t}dt}=t^{-2}\quad y=t^2\left[\int t^{-2}.(-t^2\cos t)dt\right]$

$y=t^2\left[-\sin t+c\right]\quad \underline{y=-t^2\sin t+t^2c} \Rightarrow$ g.ç

4- $(y-y^2\ln x)dx=x\ln x\,dy$

$\frac{dy}{dx}=\frac{y}{x\ln x}-\frac{y^2\cancel{\ln x}}{x\cancel{\ln x}}$

$\frac{dy}{dx}-\frac{1}{x\ln x}.y=-\frac{1}{x}.y^2 \Rightarrow$ Bernoulli

$z=\frac{1}{y}\quad \frac{dz}{dx}=\frac{-1}{y^2}.\frac{dy}{dx}$

$\underbrace{-\frac{1}{y^2}.\frac{dy}{dx}}+\frac{1}{x\ln x}.\frac{1}{y}=\frac{1}{x}$

$\frac{dz}{dx}+\frac{1}{x\ln x}.z=\frac{1}{x}$ ... L.D.D

$P(x)=\frac{1}{x\ln x}$

$Q(x)=\frac{1}{x}$

$\mu=\ln x\quad z=\frac{\ln x}{2}+\frac{c}{\ln x}$

$z=\frac{1}{y}$ idi

$y=\frac{1}{\frac{\ln x}{2}+\frac{c}{\ln x}}$ g.ç

$$y.e^{2xy}dx + 2bxe^{2xy}dy = -xdx$$

$$ye^{2xy}dx + xdx + 2bxe^{2xy}dy = 0$$

$$\underbrace{(ye^{2xy}+x)}_{P}dx + \underbrace{2bxe^{2xy}}_{Q}dy = 0$$

a) $b \in \mathbb{R}$'nin hangi değerleri için tam D.D olur?

b) Bu b değeri için dif. denk.i çözünüz.

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$ ise tam dif. denk.dır.

$$e^{2xy} + y.2x.e^{2xy} = 2b.e^{2xy} + 2bx.2y.e^{2xy}$$

$$\cancel{e^{2xy}}(\cancel{1+2xy}) = 2b\cancel{e^{2xy}}(\cancel{1+2xy})$$

$$b = \frac{1}{2}$$

$$ye^{2xy}dx + xe^{2xy}dy = -xdx$$

$$(ye^{2xy}+x)dx + xe^{2xy}dy = 0$$

$$\frac{\partial u}{\partial x} = ye^{2xy}+x, \quad \frac{\partial u}{\partial y} = xe^{2xy}$$

$$u = \int_x (ye^{2xy}+x)\,dx + R(y) = \frac{e^{2xy}}{2} + \frac{x^2}{2} + R(y)$$

$$\frac{\partial u}{\partial y} = xe^{2xy} = xe^{2xy} + \frac{dR}{dy} \Rightarrow \frac{dR}{dy} = 0 \Rightarrow \underline{R(y) = k = \text{sbt}}$$

$$u = \frac{e^{2xy}}{2} + \frac{x^2}{2} + k = c$$

$$\frac{e^{2xy}}{2} + \frac{x^2}{2} = c - k$$