

Teorem 2

$$\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1(i-1)} & a_{1i}+b_i & a_{1(i+1)} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2(i-1)} & a_{2i}+b_i & a_{2(i+1)} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \dots & a_{n(i-1)} & a_{ni}+b_i & a_{n(i+1)} & \dots & a_{nn} \end{vmatrix}$$

$$= \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1(i-1)} & a_{1i} & a_{1(i+1)} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2(i-1)} & a_{2i} & a_{2(i+1)} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \dots & a_{n(i-1)} & a_{ni} & a_{n(i+1)} & \dots & a_{nn} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1(i-1)} & b_i & a_{1(i+1)} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2(i-1)} & b_i & a_{2(i+1)} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \dots & a_{n(i-1)} & b_i & a_{n(i+1)} & \dots & a_{nn} \end{vmatrix}$$

Teorem 2 $|AB| = |A| \cdot |B|$

Teorem 2 A^{-1} mevcut ise $AA^{-1} = A^{-1}A = I \Rightarrow |A||A^{-1}| = |A^{-1}||A| = |I|$ mevcut ve

$$|A^{-1}| = \frac{1}{|A|} = |A|^{-1} \quad |A| \neq 0 \quad / \quad \text{matrisin tersi olması için gerek ve yeter şart } \rightarrow |A| \neq 0$$

Örnek 2

$$|A| = \begin{vmatrix} a & a+b & b+c \\ b & b+c & c+a \\ c & c+a & a+b \end{vmatrix}$$

det. özelliklerinden yararlanarak hesaplayınız.

$$= \begin{vmatrix} a & a+b & b \\ b & b+c & c \\ c & c+a & a \end{vmatrix} + \begin{vmatrix} a & a+b & c \\ b & b+c & a \\ c & c+a & b \end{vmatrix} = \begin{vmatrix} a & a & b \\ b & b & c \\ c & c & a \end{vmatrix} + \begin{vmatrix} a & b & b \\ b & c & c \\ c & a & a \end{vmatrix} + \begin{vmatrix} a & a & c \\ b & b & a \\ c & c & b \end{vmatrix} + \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$= \begin{vmatrix} a+b+c & b & c \\ a+b+c & c & a \\ a+b+c & a & b \end{vmatrix} = (a+b+c) \begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix} = (a+b+c) \begin{vmatrix} 0 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix} = (a+b+c) \begin{vmatrix} c-b & a-c \\ a-b & b-c \end{vmatrix}$$

2 ve 3'ü topluya
1'e ekledik

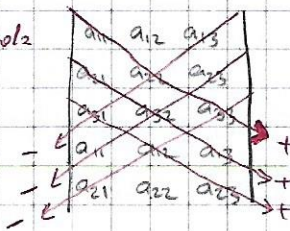
$$= (a+b+c) [(c-b)(b-c) - (a-c)(a-b)] = 3abc - a^3 - b^3 - c^3$$

Sarrus Kuralı

3x3 mertebesinde determinant hesaplarında kullanılır.

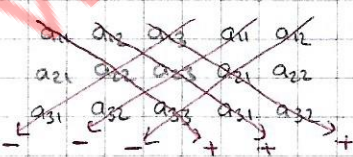
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

1. yol



$$= [(a_{11} \cdot a_{22} \cdot a_{33}) + (a_{21} \cdot a_{32} \cdot a_{13}) + (a_{31} \cdot a_{12} \cdot a_{23})] - [(a_{13} \cdot a_{22} \cdot a_{31}) + (a_{23} \cdot a_{32} \cdot a_{11}) + (a_{33} \cdot a_{12} \cdot a_{21})]$$

2. yol



Örnek 2

$$|A| = \begin{vmatrix} -1 & 4 & 2 \\ 2 & -5 & 7 \\ 6 & 2 & 0 \\ -1 & 4 & 2 \\ 2 & -5 & 7 \end{vmatrix} = [2^3 + 6 \cdot 4 \cdot 7] - [2 \cdot (-5) \cdot 6 + 7 \cdot 2 \cdot (-1)] = 250$$

Bir Matrisin Ters

Tanınır Bir kare matrisde, matrisin elemanların yerine eş karşılıklarının yazılmasıyla elde edilen matrisin transpozese "ek matris" ve "adjoint matris" denir.

$$EK(A), \text{adj}(A), A^*$$

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} \quad EK(A) = \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1n} \\ A_{21} & A_{22} & \dots & A_{2n} \\ \dots & \dots & \dots & \dots \\ A_{n1} & A_{n2} & \dots & A_{nn} \end{bmatrix}^T$$

$$= \begin{bmatrix} A_{11} & A_{21} & \dots & A_{n1} \\ A_{12} & A_{22} & \dots & A_{n2} \\ \dots & \dots & \dots & \dots \\ A_{1n} & A_{2n} & \dots & A_{nn} \end{bmatrix}$$

$$\begin{array}{l} \text{es karş.} \\ a_{11} \rightarrow A_{11} \\ a_{21} \rightarrow A_{21} \end{array}$$

Teorem 2 A bir kare matris olsun. Eğer $|A| \neq 0$ ise $A^{-1} = \frac{EK(A)}{|A|}$ 'dir.

Örnek 2 $A = \begin{bmatrix} 2 & 1 & -3 \\ 4 & -2 & 2 \\ 1 & 0 & 3 \end{bmatrix}$ matrisinin tersini ek matrisler yardımıyla hesaplayınız.

$$A = \begin{bmatrix} 2 & 1 & -3 \\ 4 & 0 & -4 \\ 1 & 0 & 3 \end{bmatrix} \quad |A| = \begin{vmatrix} 2 & 1 & -3 \\ 4 & 0 & -4 \\ 1 & 0 & 3 \end{vmatrix} = 8 - 4 = 4 \neq 0$$

$H_{21}(2)$

$$A_{11} = \begin{vmatrix} -2 & 2 \\ 0 & 3 \end{vmatrix} = -6 \quad A_{12} = - \begin{vmatrix} 4 & 2 \\ 1 & 3 \end{vmatrix} = -10 \quad A_{13} = \begin{vmatrix} 4 & -2 \\ 1 & 0 \end{vmatrix} = 2$$

$$EK(A) = \begin{bmatrix} -6 & -10 & 2 \\ -3 & 9 & 4 \\ -4 & -16 & -8 \end{bmatrix} \quad EK(A)_2 = \begin{bmatrix} -6 & -3 & -4 \\ -10 & 9 & -16 \\ 2 & 1 & -8 \end{bmatrix}$$

$$A^{-1} = -\frac{1}{28} \begin{bmatrix} -6 & -3 & -4 \\ -10 & 9 & -16 \\ 2 & 1 & -8 \end{bmatrix} = \begin{bmatrix} \frac{3}{14} & \frac{3}{28} & \frac{1}{7} \\ \frac{5}{14} & -\frac{9}{28} & \frac{4}{7} \\ -\frac{1}{14} & -\frac{1}{28} & \frac{2}{7} \end{bmatrix}$$

Soru 2 A n x n mertebesinde ters simetrik matris olsun. n'nin tek sayısı olması durumunda $|A|$ 'yı hesaplayınız.

$$A = -A^T \rightarrow \text{ters simetrik} \quad A \text{ n x n matr.} \quad |kA| = k^n |A|$$

$$|A| = |A^T|$$

$$|A| = |-A|$$

$$|A| = (-1)^n |A| = -|A| \rightarrow n \text{ tek}$$

$$2|A| = 0$$

$$|A| = 0$$

$$|A| = \begin{vmatrix} 1+x_1 & 1 & 1 & \dots & 1 \\ 1 & 1+x_2 & 1 & \dots & 1 \\ \dots & \dots & \dots & \dots & \dots \\ 1 & 1 & 1 & \dots & 1+x_n \end{vmatrix} \quad \text{det. hesaplayınız.}$$

$$|A| = \begin{vmatrix} x_1(1+\frac{1}{x_1}) & 1 & 1 & \dots & 1 \\ 1 & x_2(1+\frac{1}{x_2}) & 1 & \dots & 1 \\ \dots & \dots & \dots & \dots & \dots \\ 1 & 1 & 1 & \dots & x_n(1+\frac{1}{x_n}) \end{vmatrix} = x_1 \cdot x_2 \cdot \dots \cdot x_n \begin{vmatrix} (1+\frac{1}{x_1}) & \frac{1}{x_1} & \dots & \dots & \frac{1}{x_1} \\ \frac{1}{x_1} & (1+\frac{1}{x_2}) & \dots & \dots & \frac{1}{x_2} \\ \dots & \dots & \dots & \dots & \dots \\ \frac{1}{x_1} & \frac{1}{x_2} & \dots & \dots & (1+\frac{1}{x_n}) \end{vmatrix}$$

$$= \begin{vmatrix} (1+\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n}) & \frac{1}{x_2} & \dots & \frac{1}{x_n} \\ (1+\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n}) & \frac{1}{x_2} & \dots & \frac{1}{x_n} \\ \dots & \dots & \dots & \dots \\ (1+\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n}) & \frac{1}{x_2} & \dots & \frac{1}{x_n} \end{vmatrix} = (x_1 \cdot x_2 \cdot \dots \cdot x_n) (1 + \frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n}) = \begin{vmatrix} 1 & \frac{1}{x_2} & \dots & \frac{1}{x_n} \\ 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{vmatrix}$$

$$= (x_1 \cdot x_2 \cdot \dots \cdot x_n) (1 + \frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n}) \begin{vmatrix} 1 & \frac{1}{x_2} & \dots & \frac{1}{x_n} \\ 0 & 1 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 1 \end{vmatrix} = (x_1 \cdot x_2 \cdot \dots \cdot x_n) (1 + \frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n})$$

$\underbrace{\hspace{10em}}_{=1}$
 det = cgen

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