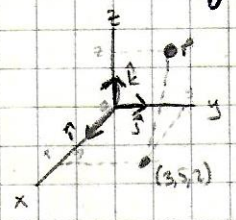
Fizik 1

Vektör

Skaler büyüklük: Sıcaklık ... Hız 10 m/s $\vec{V} = 10\hat{i}$ m/s (yön)

Vektörel büyüklük: Hız ...

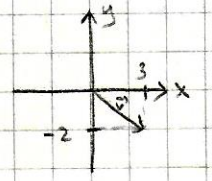


$P \rightarrow \begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$

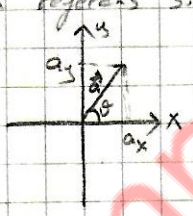
$|\hat{i}| = 1 \quad |\hat{j}| = 1 \quad |\hat{k}| = 1$

$3\hat{i} + 5\hat{j} + 2\hat{k} = \vec{r}$

Duran veya birbirine göre hareket eden referans sistemlerde fizik yasaları değişmez.



$\vec{V} = 3\hat{i} - 2\hat{j}$



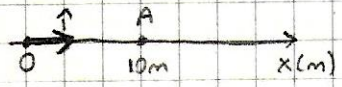
$a_x = a \cdot \cos\vartheta$

$a_y = a \cdot \sin\vartheta$

$a_x^2 + a_y^2 = a^2$

$a = \sqrt{a_x^2 + a_y^2}$

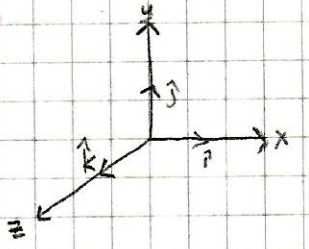
Birim Vektör



$\overrightarrow{OA} = 10\hat{i}$

$\hat{i} =$ } birim vektör

$|\hat{i}| =$



$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$

▽ Bir vektör skalerle işlem yaparsa büyüklüğü değişir.

İki Vektörün Çarpımı

$\vec{A} = Ax\hat{i} + Ay\hat{j} + Az\hat{k}$

$\vec{B} = Bx\hat{i} + By\hat{j} + Bz\hat{k}$

1- Skaler Çarpım = (Nokta) $\vec{A} \cdot \vec{B} = |\vec{A}| \cdot |\vec{B}| \cdot \cos\vartheta$ ϑ: $\vec{A}$ ile $\vec{B}$ arasındaki açı

2- Vektörel Çarpım = (Çarpı) $\vec{A} \times \vec{B} = |\vec{A}| \cdot |\vec{B}| \cdot \sin\vartheta$ (vektör)

1- Skaler Çarpım

$\vec{A}.\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}).(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$

$= (A_x\hat{i}.B_x\hat{i}) + (A_x\hat{i}.B_y\hat{j}) + (A_x\hat{i}.B_z\hat{k}) +$
$(A_y\hat{j}.B_x\hat{i}) + (A_y\hat{j}.B_y\hat{j}) + (A_y\hat{j}.B_z\hat{k}) +$
$(A_z\hat{k}.B_x\hat{i}) + (A_z\hat{k}.B_y\hat{j}) + (A_z\hat{k}.B_z\hat{k})$

$= A_xB_x(\hat{i}.\hat{i}) + A_xB_y(\hat{i}.\hat{j}) + A_xB_z(\hat{i}.\hat{k}) +$
$A_yB_x(\hat{j}.\hat{i}) + A_yB_y(\hat{j}.\hat{j}) + A_yB_z(\hat{j}.\hat{k}) +$
$A_zB_x(\hat{k}.\hat{i}) + A_zB_y(\hat{k}.\hat{j}) + A_zB_z(\hat{k}.\hat{k})$ $= A_xB_x + A_yB_y + A_zB_z$

$\hat{i}.\hat{i} = |\hat{i}|.|\hat{i}|.\cos 0 = 1$ $\qquad$ $\hat{i}.\hat{j} = |\hat{i}|.|\hat{j}|.\sin 0 = 0$

$\hat{i}.\hat{i} = 1$ $\qquad$ $\hat{i}.\hat{j} = 0$
$\hat{j}.\hat{j} = 1$ $\qquad$ $\hat{i}.\hat{k} = 0$
$\hat{k}.\hat{k} = 1$ $\qquad$ $\hat{j}.\hat{k} = 0$

Örnek: $\vec{A} = 5\hat{i} - 2\hat{j}$, $\vec{B} = 4\hat{i} + 3\hat{j}$ $\rightarrow$ $\vec{A}.\vec{B} = 20 - 6 = 14$

Skaler Çarpımın Özellikleri

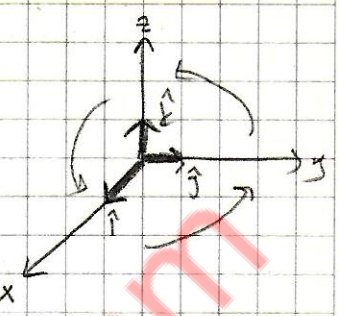
1- $\vec{A}.\vec{B} = \vec{B}.\vec{A}$

2- $\vec{A}.(\vec{B}+\vec{C}) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$

3- $m\,(\vec{A}.\vec{B}) = (m\vec{A}).\vec{B} = \vec{A}.(m\vec{B}) = (\vec{A}.\vec{B})\,m$ (m: skaler)

4- Eğer $\vec{A}.\vec{B} = 0$ ve $|\vec{A}| \neq 0$, $|\vec{B}| \neq 0$ ise $\vartheta = 90^\circ \rightarrow \vec{A} \perp \vec{B}$

2- Vektörel Çarpım



$$\vec{A}\times\vec{B} = (A_x\hat{i}+A_y\hat{j}+A_z\hat{k})\times(B_x\hat{i}+B_y\hat{j}+B_z\hat{k})$$

$$= A_xB_x(\hat{i}\times\hat{i}) + A_xB_y(\hat{i}\times\hat{j}) + A_xB_z(\hat{i}\times\hat{k}) + A_yB_x(\hat{j}\times\hat{i}) + A_yB_y(\hat{j}\times\hat{j}) + A_yB_z(\hat{j}\times\hat{k}) + A_zB_x(\hat{k}\times\hat{i}) + A_zB_y(\hat{k}\times\hat{j}) + A_zB_z(\hat{k}\times\hat{k})$$

$\hat{i}\times\hat{i} = |\hat{i}|\cdot|\hat{i}|\cdot\sin 0 = 0$, $\quad \hat{i}\times\hat{j} = |\hat{i}|\cdot|\hat{j}|\cdot\sin 90 = 1$

$\hat{i}\times\hat{i}=0$, $\hat{j}\times\hat{j}=0$, $\hat{k}\times\hat{k}=0$

$\hat{i}\times\hat{j}=\hat{k}$, $\hat{i}\times\hat{k}=-\hat{j}$, $\hat{j}\times\hat{k}=\hat{i}$

$\hat{k}\times\hat{i}=\hat{j}$, $\hat{k}\times\hat{j}=-\hat{i}$, $\hat{j}\times\hat{i}=-\hat{k}$

$$= A_xB_y(\hat{k}) + A_xB_z(-\hat{j}) + A_yB_x(-\hat{k}) + A_yB_z(\hat{i}) + A_zB_x(\hat{j}) + A_zB_y(-\hat{i})$$

$$\vec{C} = (A_yB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$$

$$\vec{C} = \underbrace{(A_yB_z - A_zB_y)\hat{i}}_{\vec{C}_x} - \underbrace{(A_xB_z - A_zB_x)\hat{j}}_{-\vec{C}_y} + \underbrace{(A_xB_y - A_yB_x)\hat{k}}_{\vec{C}_z}$$

yada

$$\vec{A}\times\vec{B} = \begin{vmatrix} +\hat{i} & -\hat{j} & +\hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\vec{C} = \underbrace{(A_yB_z - A_zB_y)\hat{i}}_{\vec{C}_x} - \underbrace{(A_xB_z - B_xA_z)\hat{j}}_{\vec{C}_y} + \underbrace{(A_xB_y - A_yB_x)\hat{k}}_{\vec{C}_z}$$

! $\vec{C}_y$ ters

Vektörel Çarpım Özellikleri

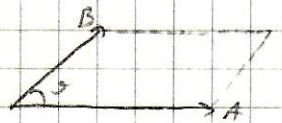
1- $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$

2- $\vec{A}\times(\vec{B}+\vec{C}) = \vec{A}\times\vec{B} + \vec{A}\times\vec{C}$

3- $m(\vec{A}\times\vec{B}) = (m\vec{A})\times\vec{B} = \vec{A}\times(m\vec{B}) = (\vec{A}\times\vec{B})m$ (m skaler)

4- $\vec{A}\times\vec{B}$ = Kenarları A ve B olan paralelkenarın alanı

5- Eğer $|\vec{A}|\neq 0$, $|\vec{B}|\neq 0$ ve $\vec{A}\times\vec{B}=0$ ise $\vec{A}\parallel\vec{B}$



Alan = $AB\sin\theta = \vec{A}\times\vec{B}$

Örnek = $\vec{m} = 6\hat{i} + 2\hat{j} - \hat{k}$, $\vec{N} = 2\hat{i} - \hat{j} - 3\hat{k}$

a) $\vec{m} \cdot \vec{N} = ?$
b) $\vec{m} \times \vec{N} = ?$
c) $\vartheta = ?$

a) $= \vec{m} \cdot \vec{N} = |\vec{m}| \cdot |\vec{N}| \cdot \cos\vartheta = 12 - 2 + 3 = 13$ br

b) $= \vec{m} \times \vec{N} = |\vec{m}| \cdot |\vec{N}| \cdot \sin\vartheta = -6 \cdot (\hat{k}) - 18 \cdot (-\hat{j}) + 4 \cdot (-\hat{k}) - 6(\hat{i}) - 2(\hat{j}) + (-\hat{i})$

$-6k - 4k + 18\hat{j} + 2\hat{j} - 6\hat{i} - \hat{i}$

$= -7\hat{i} + 16\hat{j} - 10\hat{k}$

c) $= \vec{m} \cdot \vec{N} = |\vec{m}| \cdot |\vec{N}| \cdot \cos\vartheta \rightarrow \cos\vartheta = \dfrac{\vec{m} \cdot \vec{N}}{|\vec{m}| \cdot |\vec{N}|} = \dfrac{13}{\sqrt{41} \cdot \sqrt{14}}$ $\vartheta = \cos^{-1}\left(\dfrac{13}{\sqrt{41} \cdot 14}\right)$

$|\vec{m}| = \sqrt{6^2 + 2^2 + 1} = \sqrt{41}$
$|\vec{N}| = \sqrt{2^2 + 1 + 3^2} = \sqrt{14}$