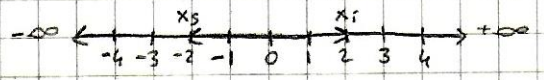
Bir Boyutta Hareket

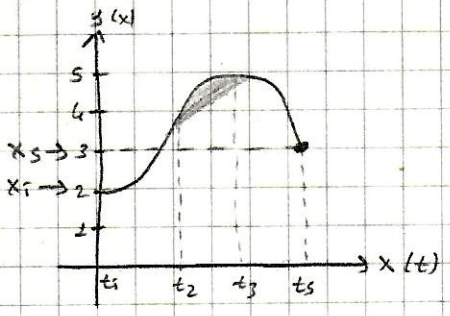
1- Yerdeğiştirme
2- Hız, Sürat
3- İvme



$x_s - x_i = -2 - 2 = -4 = \Delta x$

1- Yerdeğiştirme $= x_s - x_i = \Delta x$

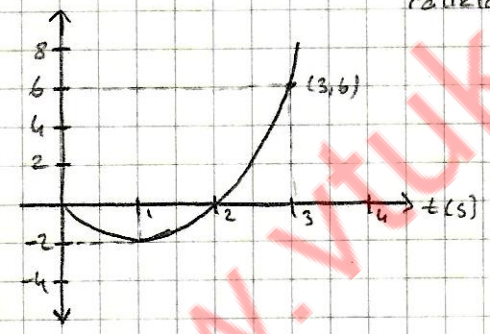
2- Hız $= \bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{x_s - x_i}{t_s - t_i} \rightarrow$ vektörel



$\bar{v} = \dfrac{x_s - x_i}{t_s - t_i}$ $\quad v = \lim_{\Delta t \to 0} \bar{v} = \lim_{\Delta t \to 0} \dfrac{\Delta x}{\Delta t}$

$v = \dfrac{dx}{dt}$ ani hız.

Örnek. $x = -4t + 2t^2$ a) $t=0$ ile $t=1$ s ve $t=1$ s ile $t=3$ s zaman aralıklarında parçacığın yerdeğiştirmesini bulunuz.



$t_s = 1$, $t_i = 0$ $\quad \Delta t = t_s - t_i = 1 - 0 = 1$

$x_i = -4(t_i) + 2(t_i)^2 = 0$ $\quad \Delta x = -2$ m

$x_s = -4(t_s) + 2(t_s)^2 = -2$

$t_s = 3$, $t_i = 1$ $\quad x_i = -4(t_i) + 2(t_i)^2 = -2$ $\quad \Delta x = 8$ m

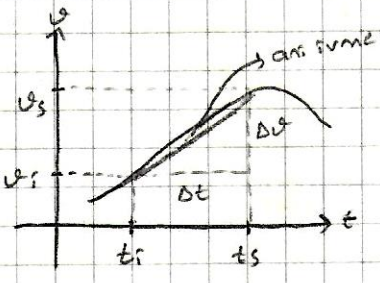
$x_s = -4(t_s) + 2(t_s)^2 = 6$

b) $t=0$ ile $t=1$
$t=1$ ile $t=3$ ortalama hızları bulunuz.

$0\text{-}1 \rightarrow \bar{v}_{01} = \dfrac{\Delta x}{\Delta t} = \dfrac{-2}{1} = -2$ m/s

$1\text{-}3 \rightarrow \bar{v}_{13} = \dfrac{\Delta x}{\Delta t} = \dfrac{+8}{2} = 4$ m/s

3- İvme $= \bar{a} = \dfrac{v_s - v_i}{t_s - t_i}$ $\quad \left(\dfrac{m/s}{s}\right) \Rightarrow m/s^2 = \dfrac{\Delta v}{\Delta t}$



ani ivme $\quad a = \lim_{\Delta t \to 0} \bar{a} = \lim_{\Delta t \to 0} \dfrac{\Delta v}{\Delta t} = \dfrac{dv}{dt}$

$\Delta x = x_s - x_i$

$\bar{v} = \dfrac{\Delta x}{\Delta t}$ $\quad v = \dfrac{dx}{dt}$

$\bar{a} = \dfrac{\Delta v}{\Delta t}$ $\quad a = \dfrac{dv}{dt}$ $\quad a = \dfrac{d}{dt}\left(\dfrac{dx}{dt}\right) = \dfrac{d^2x}{dt^2}$ $\quad$ Jerk $= \dfrac{da}{dt} = J$ (sarsıntı)

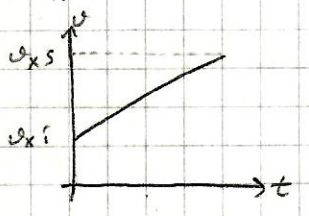
Bir Boyutta Sabit İvmeli Hareket

$a_x = \bar{a}_x$ = sabit (zamanla değişmiyor)

$t_i = 0$, $t_s = t$

$\bar{a} = \frac{v_s - v_i}{t_s - t_i}$ $\quad a_x = \frac{v_{xs} - v_{xi}}{t - 0}$

a_x = sabit = $\bar{a}_x$



$v_{xs} - v_{xi} = a_x t$ $\quad$ **$v_{xs} = v_{xi} + a_x t$**

$\bar{v}_x = \frac{x_s - x_i}{t_s - t_i} = \frac{x_s - x_i}{t}$ $\quad x_s - x_i = \bar{v}_x t$

$x_s - x_i = \left(\frac{v_{xs} + v_{xi}}{2}\right) \cdot t$

$\bar{v}_x = \frac{v_{xs} + v_{xi}}{2}$ (sbt a_x için)

$x_s - x_i = \left(\frac{v_{xi} + a_x t + v_{xi}}{2}\right) \cdot t$

$\Delta x = x_s - x_i = v_{xi} t + \frac{1}{2} a_x t^2$

$x_s = x_i + v_{xi} t + \frac{1}{2} a_x t^2$ → Parçacığın son konumu ifadesi

$x_s - x_i = \left(\frac{v_{xs} + v_{xi}}{2}\right) \cdot t$ $\quad v_{xs} = v_{xi} + a_x t$ idi, buradan $t = \frac{v_{xs} - v_{xi}}{a_x}$

$x_s - x_i = \frac{1}{2}(v_{xs} + v_{xi})\left(\frac{v_{xs} - v_{xi}}{a_x}\right)$

$2 a_x \underbrace{(x_s - x_i)}_{\Delta x} = v_{xs}^2 - v_{xi}^2$ $\quad$ " $\Delta x = \frac{v_{xs}^2 - v_{xi}^2}{2 a_x}$ "

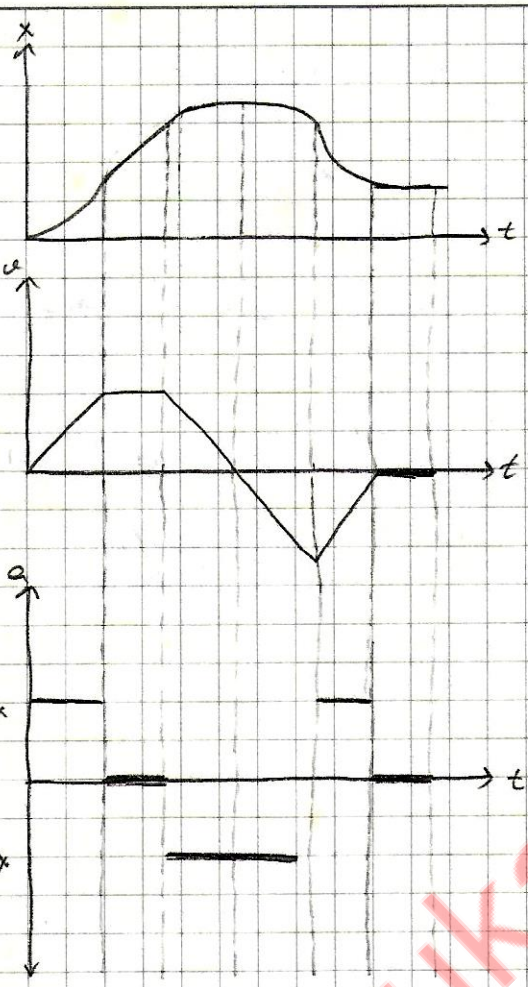
$v_{xs}^2 = v_{xi}^2 + 2 a_x (\Delta x)$ → zamansız hız denklemi

Örnek: Bir jet 63 m/s hızla 2s'de iniyor.

a) Jet 2s sonra <u>duruyor</u> ise ivmesi?

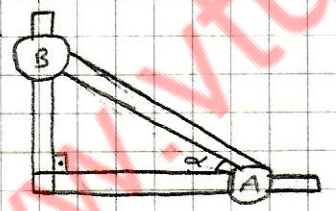
$a_x = \frac{v_{xs} - v_{xi}}{t_s - t_i} = \frac{0 - 63}{2} = -31{,}5 \ m/s^2$

b) $\Delta x = \frac{v_{xs}^2 - v_{xi}^2}{2 a_x} \rightarrow \frac{0 - (63)^2}{2 \cdot (-31{,}5)}$ $\quad \frac{-63 \cdot 63}{-63} = \frac{63 \ m}{}$



a_x $x=?$ $v_x=?$

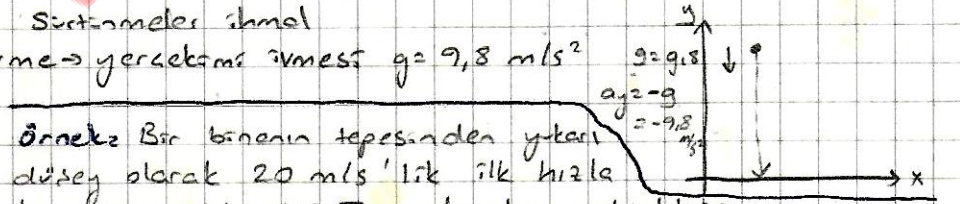
Örnek 2 A cismi sbt v hızı ile sola kayar ise $\alpha=60°$ olduğu anda B'nin hızı=?



Serbest Düşen Cisimler

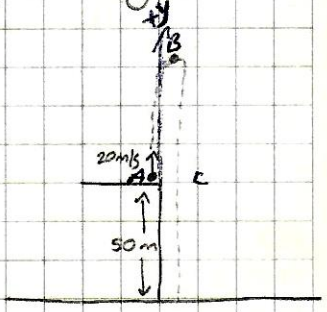
Sürtünmeler ihmal

ivme → yerçekimi ivmesi $g = 9{,}8\ m/s^2$



Örnek: Bir binanın tepesinden yukarı düşey olarak 20 m/s'lik ilk hızla bir taş atılmıştır. Taş düşerken yüksekliği 50 m olan binanın çatısını şekilde gösterildiği gibi sıyırarak geçer. Taşın atıldığı (A) noktasında $t_A = 0$ seçerek

a) taşın max yüksekliğine ulaştığı zaman?
b) max yüksekliği
c) taşın atıldığı noktaya geri dönüş zamanını?
d) taşın bu andaki hızını?
e) t_2'de taşın hızını ve konumunu bulunuz.



a) $v_{yB} = v_{yA} + a_y t$

$v_{yB} = v_{yA} - g_y t$

$0 = 20\,m/s + (9,8)t$

$t = \frac{20}{9,8} = 2,04\,s$

b) $y_s - y_i = v_{yi} t + \frac{1}{2} a_y t^2$

$y_s = y_i + v_{yi} t + \frac{1}{2} a_y t^2$

$y_s = 0 + (20\,m/s)(2,04) + \frac{1}{2}(-9,8)(2,04)^2$

$y_s = 20,4\,m$

Zemine göre $\rightarrow y_z = y_s + 50$

$70,4\,m$

c) $y_s = y_i + v_{yi} t + \frac{1}{2} a_y t^2$

$0 = 0 + (20\,m/s)t + \frac{1}{2}(-9,8)t^2$

$\frac{9,8t^2}{2} = 20t \qquad t_1 = 0 \qquad t_2 = 4,08\,s$

d) $v_{ys} = v_{yi} + a_y t$

$v_{ys} = (20\,m/s) + (-9,8)(4,08)$

$v_{ys} = -20\,m/s$

e) $t = 5$

$y_s = y_i + v_{yi} t + \frac{1}{2} a_y t^2$

$= 0 + (20).(5) + \frac{1}{2}(-9,8).25$

$= +29,016\,m$

$v_{ys} = v_{yi} + a_y t$

$= v_{yi} - g_y t$

$v_{ys} = 20 - 9,8.5$

$= -29,5\,m/s$

Kinematik Denklemlerin Matematiksel Yöntemle Türetilmesi

$a_x = \frac{dv_x}{dt} \Rightarrow \int_{v_i}^{v_s} dv_x = \int_{t_i}^{t_s} a_x\,dt$

ivme sabit

$v_x \Big|_{v_i}^{v_s} = a_x t \Big|_{t_i}^{t_s}$

$v_s - v_i = a_x (t_s - t_i)$

$v_s = v_i + a_x (t_s - t_i)$

$v_x = \frac{dx}{dt} \qquad dx = v_x . dt$

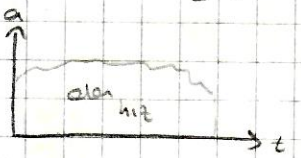
$\int_{x_i}^{x_s} dx = \int_{t_i}^{t_s} v_x\,dt$

$\int dx = \int_0^t (v_{xi} + a_x t)\,dt$

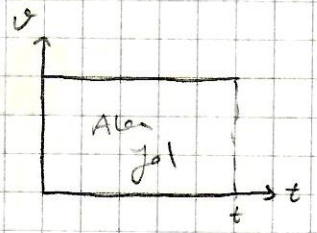
$x \Big|_{x_i}^{x_s} = v_{xi} t + \frac{1}{2} a_x t^2$

$v_x = v_{xs} = v_{xi} + a_x t$

$x_s - x_i = v_{xi} t + \frac{1}{2} a_x t^2$

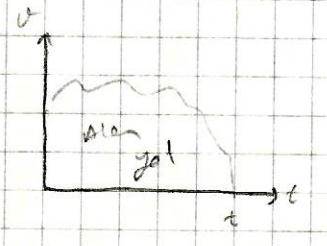


$v = \int a(t)\,dt$



$v = \frac{x}{t}$

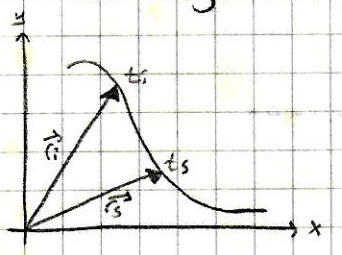
$x = vt$



$x = \int v(t)\,dt$

İki Boyutlu Hareket

Yer değiştirme, hız ve ivme vektörleri



yer vektörü = $\vec{r}$ = konum vektörü → orijinden

$\vec{r}_i$ = ilk konum vektörü

$\vec{r}_s$ = son konum vektörü

$$\Delta\vec{r} = \vec{r}_s - \vec{r}_i \qquad \vec{r}_s = \vec{r}_i + \Delta\vec{r}$$

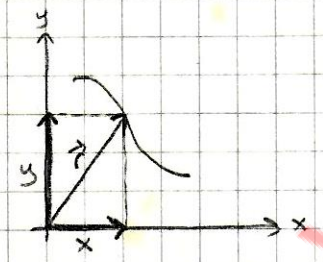
→ yerdeğiştirme vektörü

$$\vec{v} = \frac{\Delta\vec{r}}{\Delta t} \ (m/s)$$ ortalama hız vektörü

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} \ (m/s)$$ anlık hız vektörü

$$\vec{a} = \frac{\vec{v}_s - \vec{v}_i}{t_s - t_i} \qquad \vec{a} = \frac{\Delta\vec{v}}{\Delta t} \ (m/s^2)$$ ortalama ivme vektörü

$$\vec{a} = \lim_{\Delta t \to 0} \frac{d\vec{v}}{dt} \Rightarrow \vec{a} = \frac{d\vec{v}}{dt} \ (m/s^2)$$ anlık ivme vektörü



$\vec{r} = x\hat{i} + y\hat{j}$

$\vec{v} = \frac{d\vec{r}}{dt}$

hız $\vec{v} = \frac{d}{dt}(x\hat{i} + y\hat{j})$

$\vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$

$\vec{v} = v_x\hat{i} + v_y\hat{j}$

ivme $\vec{a} = \frac{d\vec{v}}{dt}$ $\qquad \vec{a} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j}$

$\vec{a} = \frac{d}{dt}(v_x\hat{i} + v_y\hat{j})$ $\qquad \vec{a} = a_x\hat{i} + a_y\hat{j}$

a_x ve a_y sabit olsun

her bir eksen sabit ivmeli bir boyutlu hareket olarak göz önüne alınabilir.

$v_{xs} = v_{xi} + a_x t$

$v_{ys} = v_{yi} + a_y t$

$\vec{v}_s = v_{xs}\hat{i} + v_{ys}\hat{j}$

$\vec{v}_s = (v_{xi} + a_x t)\hat{i} + (v_{yi} + a_y t)\hat{j}$

$\vec{v}_s = \underbrace{v_{xi}\hat{i} + v_{yi}\hat{j}}_{\vec{v}_i} + a_x t\hat{i} + a_y t\hat{j}$

$x_s = x_i + v_{xi}t + \frac{1}{2}a_x t^2$

$y_s = y_i + v_{yi}t + \frac{1}{2}a_y t^2$

$\vec{r}_s = x_s\hat{i} + y_s\hat{j}$

$\vec{r}_s = (x_i + v_{xi}t + \frac{1}{2}a_x t^2)\hat{i} + (y_i + v_{yi}t + \frac{1}{2}a_y t^2)\hat{j}$

$= \underbrace{x_i\hat{i} + y_i\hat{j}}_{\vec{r}_i} + \underbrace{(v_{xi}\hat{i} + v_{yi}\hat{j})}_{\vec{v}_i}t + \underbrace{\frac{1}{2}(a_x\hat{i} + a_y\hat{j})}_{\vec{a}}t^2$

$\vec{v}_s = \vec{v}_i + (\underbrace{a_x\hat{i} + a_y\hat{j}}_{\vec{a}})t$

$\vec{v}_s = \vec{v}_i + \vec{a}t$

$$\vec{r}_s = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$

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$v_{yB} = v_{yA} - g_y t$

$0 = 20\ m/s - (9,8)t$

$t = \frac{20}{9,8} = 2,04\ s$

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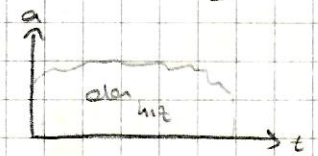
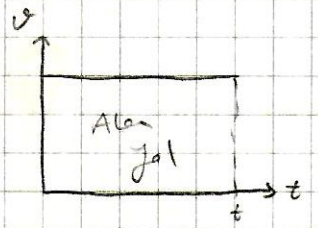
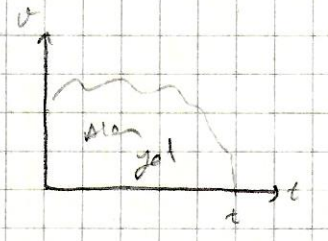


$v = \int a(t)\, dt$



$v = \frac{x}{t}$

$x = vt$



$\lambda = \int v(t)\, dt$