4) Consider the differential equation $(y-xy')\left(x-\frac{y}{y'}\right)=-2$.

Find the general solution of this equation and the singular solution of the this equation in cartesian form.

(Here $y'>0$)

$$(y-xy')(xy'-y)=-2y'$$

$$(y-xy')^2=2y'$$

$$y-xy'=\pm\sqrt{2}\sqrt{y'} \qquad y=xy'\pm\sqrt{2}\sqrt{y'} \quad \text{Clairaut d.d.}$$

$$y'=p \qquad y=xp\pm\sqrt{2}\sqrt{p}$$

$$p=p+xp'\pm\frac{\sqrt{2}}{2\sqrt{p}}p'$$

$$0=p'\left(x\pm\frac{\sqrt{2}}{2\sqrt{p}}\right)$$

$$p'=0 \Rightarrow p=c \Rightarrow y=xc\pm\sqrt{2}\sqrt{c} \quad \text{G.Ç}$$

$$x\pm\frac{\sqrt{2}}{2\sqrt{p}}=0 \Rightarrow x=\mp\frac{\sqrt{2}}{2\sqrt{p}}$$

$$y=\mp\frac{\sqrt{2}}{2\sqrt{p}}\cdot p\pm\sqrt{2}\sqrt{p}=\pm\frac{\sqrt{2}}{2}\sqrt{p}$$

$$xy=-\frac{1}{2} \quad \text{T.Ç}$$

$y'=p=$

$(\;)^2 \Rightarrow$

YTU – Faculty of Arts and Sciences
1st Midterm Exam Questions and Solutions Sheet

Name-Surname			Score Table		
Number		Group No			TOTAL
Department			Date	04/10/2017	
Course	MAT2411 Differential Equations		Duration	90dk	Room
Lecturer			Signature		

1-) Find the general solution of the differential equation $(1-\sqrt{3})y'+y\,Secx=y^{\sqrt{3}}Secx$

$$y^{-\sqrt{3}} \,/\, (1-\sqrt{3})y'+y\cdot Secx=y^{\sqrt{3}}\cdot Secx \qquad \text{Bernoulli d.d}$$

$$(1-\sqrt{3})y^{-\sqrt{3}}\cdot y'+y^{1-\sqrt{3}}\cdot Secx=Secx$$

$$y^{1-\sqrt{3}}=u$$

$$(1-\sqrt{3})y^{-\sqrt{3}}\cdot y'=u'$$

$$u'+Secx\cdot u=Secx \qquad \text{Lineer d.d} \rightarrow$$

$$\lambda=e^{\int secx\,dx}=Secx+tanx$$

$$(Secx+tanx)u'+(Sec^2x+tanx\,Secx)u=Sec^2x+Secx\,tanx$$

$$\frac{d}{dx}\left((Secx+tanx)\cdot u\right)=Sec^2x+Secx\,tanx$$

$$\int d\left((Secx+tanx)u\right)=\int\left(Sec^2x+Secx\cdot tanx\right)dx$$

$$(Secx+tanx)\cdot u=tanx+Secx+C$$

$$u=1+\frac{c}{Secx+tanx}=y^{1-\sqrt{3}}$$

Good Luck

3) Show that the differential equation $y^2dx+(3xy-e^y)dy=0$ is not exact. Solve the given equation by finding an integrating factor.

$$\underbrace{y^2}_{P}dx+\underbrace{(3xy-e^y)}_{Q}dy=0$$

$$\frac{\partial P}{\partial y}=2y \neq \frac{\partial Q}{\partial x}=3y \quad \text{d.d. tam değil}$$

$$\lambda(y)=e^{\int\frac{Q_x-P_y}{P}dy}=e^{\int\frac{3y-2y}{y^2}dy}=e^{\int\frac{dy}{y}}=y$$

$$y^3dx+(3xy^2-ye^y)dy=0$$

$$\frac{\partial u}{\partial x}=y^3 \qquad \frac{\partial u}{\partial y}=3xy^2-y.e^y \qquad du(x,y)=0 \qquad u(x,y)=C \quad \text{G.Ç}$$

$$\frac{\partial u}{\partial x}=y^3 \to u(x,y)=\int y^3dx+H(y) \Rightarrow u(x,y)=y^3x+H(y)$$

$$\frac{\partial u}{\partial y}=3y^2x+H'(y)=3xy^2-y.e^y \to H'(y)=-y.e^y$$

$$H(y)=-\int y.e^y dy=-[y.e^y-e^y]+k \qquad H(y)=-ye^y+e^y+k$$

$$u(x,y)=y^3x-ye^y+e^y+k=c$$

$$y^3x-ye^y+e^y=C$$

2) a) Find the differential equation of the family of curves $c_1(x+2)+c_2(x+2)\ln(x+2)=y$ $\quad c_1,c_2\in R^+$

$$c_1(x+2)+c_2(x+2).\ln(x+2)=y$$

$$(x+2)\ / \quad c_1+c_2.\ln(x+2)+c_2.(x+2)\frac{1}{(x+2)}=y'$$

$$(x+2)^2\ / \quad c_2\frac{1}{x+2}=y''$$

$$c_1(x+2)+c_2(x+2).\ln(x+2)=y$$

$$c_1(x+2)+c_2(x+2).\ln(x+2)+c_2(x+2)=(x+2)y'$$

$$c_2(x+2)=(x+2)^2y''$$

$$y+(x+2)^2y''=(x+2)y'$$

$$(x+2)^2y''-(x+2)y'+y=0$$

b) Find the general solution of the differential equation $y(x+3)dx+(x+2)(ydx-xdy)=0$.

$$y(x+3)dx+(x+2)y\,dx-x(x+2)dy=0$$

$$y(2x+5)dx=x(x+2)dy$$

$$\int\frac{(2x+5)dx}{x(x+2)}=\int\frac{dy}{y}$$

$\frac{A}{x}+\frac{B}{x+2}=\frac{2x+5}{x(x+2)}$

$A+B=2$

$2A=5$

$B=2-\frac{5}{2}=-\frac{1}{2}$

$$\int\left[\frac{5/2}{x}+\frac{-1/2}{x+2}\right]dx=\ln|y|+\ln|c|$$

$$\frac{5}{2}\ln|x|-\frac{1}{2}\ln|x+2|=\ln|y|+\ln|c|$$

$$\ln\sqrt{\frac{x^5}{x+2}}-\ln|y|=\ln|c|$$

$$\frac{x^5}{y^2.(x+2)}=k$$