Bazı Elementer Fonk. Laplace Dönüşümü

1) $f(t)=c=\text{sbt}$ $\qquad \mathcal{L}\{c\}=?$

$$\mathcal{L}\{c\}=\int_0^\infty \underset{\nwarrow c}{f(t)}.e^{-st}.dt=\lim_{A\to\infty}\int_0^A c.e^{-st}.dt=\lim_{A\to\infty}\left.\frac{c.e^{-st}}{-s}\right|_0^A=\lim_{A\to\infty}\left[\underbrace{-\frac{c}{s}}_{\text{sayı}}.\cancel{e^{-sA}}-\left(-\frac{c}{s}\right).e^0\right]=\frac{c}{s}$$

$$\frac{1}{e^{sA}}=0$$

$\boxed{\mathcal{L}\{c\}=\frac{c}{s}}$ $\qquad \mathcal{L}\{5\}=\frac{5}{s}$ $\qquad \mathcal{L}\{9\}=\frac{9}{s}$ $\qquad \mathcal{L}\{\frac{1}{2}\}=\frac{1}{2s}$

$\mathcal{L}\{1\}=\frac{1}{s}$ $\qquad \mathcal{L}\{\sqrt{3}\}=\frac{\sqrt{3}}{s}$

2) $f(t)=t$ $\qquad \mathcal{L}\{t\}=?$

$$\mathcal{L}\{t\}=\int_0^\infty \underset{\nwarrow t}{f(t)}.e^{-st}.dt=\lim_{A\to\infty}\int_0^A t.e^{-st}.dt=\lim_{A\to\infty}\left[\left.\frac{t.e^{-st}}{-s}\right|_0^A+\frac{1}{s}\int_0^A e^{-st}.dt\right]=\lim_{A\to\infty}\left[\underbrace{-\frac{1}{s}.A.e^{-s.A}}_{\infty.0\ \text{BE}\ ??}-\cancel{\frac{0}{s}e^0}+\frac{1}{s}.\left.\frac{e^{-st}}{-s}\right|_0^A\right]$$

$\lim_{A\to\infty} A.e^{-sA}=0.\infty$ BE

$\lim_{A\to\infty}\frac{A}{e^{sA}}=\frac{\infty}{\infty}$ B.E.

$\lim_{A\to\infty}\frac{1}{s.e^{sA}}=0$

$u=t \qquad dv=e^{-st}.dt$

$du=dt \qquad v=\frac{e^{-st}}{-s}$

$$\mathcal{L}\{t\}=\lim_{A\to\infty}\left[\cancel{\frac{1}{s}.\frac{e^{-sA}}{-s}}+\frac{1}{s^2}.e^0\right]=\frac{1}{s^2}$$

$\boxed{\mathcal{L}\{t\}=\frac{1}{s^2}}$

$\mathcal{L}\{t^2\}=\frac{2}{s^3}$ $\qquad \boxed{\mathcal{L}\{t^n\}=\frac{n!}{s^{n+1}}}$ $\qquad \mathcal{L}\{t^{20}\}=\frac{20!}{s^{21}}$

$\mathcal{L}\{t^3\}=\frac{3.2}{s^4}$ $\qquad \mathcal{L}\{t^{100}\}=\frac{100!}{s^{101}}$

3) $f(t)=e^{at}$ $\quad \boxed{s>a}$ $\quad \mathcal{L}\{e^{at}\}=?$

$$\mathcal{L}\{e^{at}\}=\int_0^\infty f(t).e^{-st}.dt=\lim_{A\to\infty}\int_0^A e^{-(s-a)t}.dt=\lim_{A\to\infty}\left.\frac{e^{-(s-a)t}}{-(s-a)}\right|_0^A=\lim_{A\to\infty}\left[\cancel{\frac{e^{-(s-a)A}}{-(s-a)}}-\frac{e^0}{-(s-a)}\right]=\frac{1}{s-a}$$

$\boxed{\mathcal{L}\{e^{at}\}=\frac{1}{s-a}}$ $\qquad \mathcal{L}\{e^{5t}\}=\frac{1}{s-5}$ $\qquad \mathcal{L}\{e^{-10t}\}=\frac{1}{s-(-10)}$ $\qquad \mathcal{L}\{e^{\sqrt{3}t}\}=\frac{1}{s-\sqrt{3}}$

4) $f(t)=\sin at$

$$\mathcal{L}\{\sin at\}=\int_0^\infty \sin at.e^{-st}.dt=\frac{a}{s^2+a^2}$$

5) $f(t)=\cos at$

$$\mathcal{L}\{\cos at\}=\int_0^\infty \cos at.e^{-st}.dt=\frac{s}{s^2+a^2}$$

6) $f(t)=\sinh at$

$$\mathcal{L}\{\sinh at\}=\int_0^\infty \sinh at.e^{-st}.dt=\frac{a}{s^2-a^2}$$

$\mathcal{L}\{\sin 2t\}=\frac{2}{s^2+2^2}$

$\mathcal{L}\{\cos 2t\}=\frac{s}{s^2+2^2}$

$\mathcal{L}\{\sinh 2t\}=\frac{2}{s^2-2^2}$

$\mathcal{L}\{\cosh 2t\}=\frac{s}{s^2-2^2}$

7) $f(t)=\cosh at$

$$\mathcal{L}\{\cosh at\}=\int_0^\infty \cosh at.e^{-st}.dt=\frac{s}{s^2-a^2}$$

Laplace Dönüşümünün Özellikleri

1) Lineerlik Özelliği

$$L\{c_1f_1(t)+c_2f_2(t)+\dots+c_nf_n(t)\} = c_1L\{f_1(t)\}+c_2L\{f_2(t)\}+\dots+c_nL\{f_n(t)\}$$

★ $L\{3t^2-4\sin 3t-7e^{-2t}\} = 3L\{t^2\}-4L\{\sin 3t\}-7L\{e^{-2t}\}$

$$= \frac{3.2!}{s^3} - 4.\frac{3}{s^2+3^2} - 7.\frac{1}{s-(-2)}$$

2) Kaydırma Özelliği

$$L\{f(t)\}=F(s) \Rightarrow L\{f(t).e^{at}\} = F(s-a)$$

★ $L\{e^{5t}.t^{10}\} = ?$ ★ $L\{e^{-t}.\cos 2t\} = ?$

$L\{t^{10}\} = \frac{10!}{s^{11}}$ $L\{e^{5t}.t^{10}\} = \frac{10!}{(s-5)^{11}}$ $L\{\cos 2t\} = \frac{s}{s^2+2^2}$ $L\{e^{-t}.\cos 2t\} = \frac{s-(-1)}{(s-(-1))^2+2^2}$

3) $L\{f(t)\}=F(s) \Rightarrow L\{f'(t)\} = ?$

$$L\{f'(t)\} = \int_0^\infty f'(t).e^{-st}.dt = \lim_{A\to\infty}\int_0^A f'(t).e^{-st}.dt$$

$u=e^{-st}$	$dv=f'(t)dt$
$du=-s.e^{-st}dt$	$v=f(t)$

$$L\{f'(t)\} = \lim_{A\to\infty}\left[e^{-st}.f(t)\Big|_0^A + s\int_0^A f(t).e^{-st}.dt\right]$$

$$L\{f'(t)\} = \lim_{A\to\infty}\left[\cancel{e^{-sA}.f(A)} - e^{-s.0}.f(0)\right] + s.\underbrace{\int_0^\infty f(t).e^{-st}.dt}_{L\{f(t)\}=F(s)}$$

$F(s)$
$f(0)$
$f'(0)$
$f''(0)$
$f'''(0)$

$$L\{f'(t)\} = sF(s)-f(0)$$

$$L\{f''(t)\} = s^2F(s)-sf(0)-f'(0)$$

$$L\{f'''(t)\} = s^3F(s)-s^2f(0)-sf'(0)-f''(0)$$

$$L\{f^{(n)}(t)\} = s^nF(s)-s^{n-1}f(0)-\dots-sf^{(n-2)}(0)-f^{(n-1)}(0)$$

4) $L\{f(t)\}=F(s) \Rightarrow L\{t^n.f(t)\} = (-1)^n.\frac{d^n}{ds^n}(F(s))$

★ $L\{t.\sin 5t\} = ?$

$L\{\sin 5t\} = \frac{5}{s^2+5^2}$

$L\{t.\sin 5t\} = (-1).\left(\frac{5}{s^2+5^2}\right)' = -\left(\frac{-5.2s}{(s^2+5^2)^2}\right)$

★ $L\{t^2.e^{2t}\} = ?$

1.yol $L\{e^{2t}\} = \frac{1}{s-2}$

$L\{t^2.e^{2t}\} = (-1)^2.\frac{d^2}{ds^2}\left(\frac{1}{s-2}\right) = \frac{2}{(s-2)^3}$

2.yol 2 nci kural

$L\{t^2\} = \frac{2!}{s^3}$

$L\{t^2.e^{2t}\} = \frac{2!}{(s-2)^3}$

Ters Laplace Dönüşümü

$$\mathcal{L}\{f(t)\} = F(s) \Rightarrow \mathcal{L}^{-1}\{F(s)\} = f(t)$$

1) $F(s) = \frac{1}{s} \Rightarrow \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = 1$

$F(s) = \frac{1}{s^2} \Rightarrow \mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} = t$

$F(s) = \frac{1}{s^3} \Rightarrow \mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\} = \frac{t^2}{2!}$ $\qquad \mathcal{L}\{t^2\} = \frac{2!}{s^3}$

$F(s) = \frac{1}{s^n} \Rightarrow \mathcal{L}^{-1}\left\{\frac{1}{s^n}\right\} = \frac{t^{(n-1)}}{(n-1)!}$ $\qquad \mathcal{L}\{t^{(n-1)}\} = \frac{(n-1)!}{s^n}$

2) $F(s) = \frac{1}{s-a} \Rightarrow \mathcal{L}^{-1}\left(\frac{1}{s-a}\right) = e^{at}$ $\qquad \mathcal{L}\{t\} = \frac{1}{s^2}$

$F(s) = \frac{1}{(s-a)^2} \Rightarrow \mathcal{L}^{-1}\left\{\frac{1}{(s-a)^2}\right\} = t \cdot e^{at}$ $\qquad \mathcal{L}\{t^2\} = \frac{2!}{s^3}$

$F(s) = \frac{1}{(s-a)^3} \Rightarrow \mathcal{L}^{-1}\left\{\frac{1}{(s-a)^3}\right\} = \frac{t^2}{2!} \cdot e^{at}$ $\qquad \mathcal{L}\left\{\frac{t^2}{2!}\right\} = \frac{1}{s^3}$

$\vdots$

$F(s) = \frac{1}{(s-a)^{n+1}} \Rightarrow \mathcal{L}^{-1}\left\{\frac{1}{(s-a)^{n+1}}\right\} = \frac{t^n}{n!} \cdot e^{at}$ $\qquad \mathcal{L}\{t^n\} = \frac{n!}{3^{n+1}}$

3) $F(s) = \frac{1}{s^2+a^2} \Rightarrow \mathcal{L}^{-1}\left\{\frac{1}{s^2+a^2}\right\} = \frac{1}{a}\sin at$

$F(s) = \frac{s}{s^2+a^2} \Rightarrow \mathcal{L}^{-1}\left\{\frac{s}{s^2+a^2}\right\} = \cos at$

$F(s) = \frac{1}{s^2-a^2} \Rightarrow \mathcal{L}^{-1}\left\{\frac{1}{s^2-a^2}\right\} = \frac{1}{a}\sinh at$

$F(s) = \frac{s}{s^2-a^2} \Rightarrow \mathcal{L}^{-1}\left\{\frac{s}{s^2-a^2}\right\} = \cosh at$

Ters Laplace Dönüşümünün Özellikleri

1) Lineerlik Öz.

$$\mathcal{L}^{-1}\{c_1F_1(s) + c_2F_2(s) + \dots + c_nF_n(s)\} = c_1\mathcal{L}^{-1}\{F_1(s)\} + c_2\mathcal{L}^{-1}\{F_2(s)\} + \dots + c_n\mathcal{L}^{-1}\{F_n(s)\}$$

☆ $\mathcal{L}^{-1}\left\{\frac{4}{s-2} - \frac{3s}{s^2+16} + \frac{5}{s^2+4}\right\} = ?$ $\quad 4\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\} - 3\left\{\frac{s}{s^2+4^2}\right\} + \frac{5}{2}\cdot\mathcal{L}^{-1}\left\{\frac{1\cdot 2}{s^2+2^2}\right\}$

$$4\cdot e^{2t} - 3\cdot\cos 4t + \frac{5}{2}\cdot\sin 2t$$

2) Kaydırma Öz.

$$\mathcal{L}^{-1}\{F(s)\} = f(t) \Rightarrow \mathcal{L}^{-1}\{F(s-a)\} = e^{at}\cdot f(t)$$

☆ $\mathcal{L}^{-1}\left\{\frac{1}{s^2-2s+5+1-1}\right\} = \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1\cdot 2}{(s-1)^2+2^2}\right\} = \frac{1}{2}\cdot\sin 2t\cdot e^t$

GIPTA