Euler Diferansiyel Denklemi

$$a_0 x^n y^{(n)} + a_1 x^{n-1} y^{(n-1)} + \dots + a_{n-1} . x . y' + a_n . y = f(x)$$

$x = e^t$ dönüşüme ile çözülür $\rightsquigarrow \frac{dx}{dt} = e^t$

$$\frac{dy}{dx} = \frac{dy}{dt} . \frac{dt}{dx} = \frac{dy}{dt} . \frac{1}{\left(\frac{dx}{dt}\right)} = \frac{dy}{dt} . \frac{1}{e^t} = e^{-t} . \frac{dy}{dt} = e^{-t} D y$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(e^{-t} . \frac{dy}{dt}\right) = \frac{d}{dt}\left(e^{-t} . \frac{dy}{dt}\right) \frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \left(-e^{-t} . \frac{dy}{dt} + e^{-t} . \frac{d^2y}{dt^2}\right) . \frac{1}{\left(\frac{dx}{dt}\right)} \to e^t \qquad = e^{-2t} . \Big(\underbrace{\frac{d^2y}{dt^2}}_{D^2 y} - \underbrace{\frac{dy}{dt}}_{D y}\Big) = e^{-2t} D(D-1) y$$

$$\frac{d^3y}{dx^3} = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = \frac{d}{dx}\left(e^{-2t} . \left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)\right) = \frac{d}{dt}\left(e^{-2t} . \left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)\right) \frac{dt}{dx}$$

$$\frac{d^3y}{dx^3} = \left[-2e^{-2t}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right) + e^{-2t}\left(\frac{d^3y}{dt^3} - \frac{d^2y}{dt^2}\right)\right] . \frac{1}{\left(\frac{dx}{dt}\right)} \leftarrow e^t$$

$$\frac{d^3y}{dx^3} = e^{-3t}\left[\frac{d^3y}{dt^3} - \frac{3d^2y}{dt^2} + \frac{2dy}{dt}\right] = e^{-3t} . D . (D-1)(D-2)$$

$$D^3 y - 3D^2 y + 2Dy$$

$$D(D^2 - 3D + 2) y$$

$$\quad -2 \quad -1$$

$$\frac{d^n y}{dx^n} = e^{-nt} . D . (D-1)(D-2) \dots (D-(n-1)) . y$$

dönüşüme ile sabit katsayılı dif. denk. dönüştürülerek çözülür

Soru: $x^2 y'' - 3xy' + 4y = x + x^2 \ln x \iff (x^3 y'' - 3x^2 y' + 4xy = x^2 + x^3 \ln x)$ olsaydı x'e bölerdik.

$x = e^t \qquad D = \frac{d}{dt}$

$$\frac{dy}{dx} = e^{-t} Dy = e^{-t} \frac{dy}{dt}$$

$$\frac{d^2y}{dx^2} = e^{-2t} \underbrace{D . (D-1) y}_{D^2 y - Dy} = e^{-2t}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)$$

$$e^{2t} . e^{-2t}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right) - 3 . e^t . e^{-t} . \frac{dy}{dt} + 4y = e^t + e^{2t} \underbrace{\ln e^t}_{t}$$

$$\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 4y = \underbrace{e^{1t}}_{v_1} + \underbrace{t . e^{2t}}_{v_2}$$

$$r^2 - 4r + 4 = 0$$

$$\quad -2 \quad -2$$

$r_1 = r_2 = 2 \qquad u = (c_1 + c_2 t) e^{2t}$

$v_1 = K . e^{1t}$

Kar. denk. 1 yok. t ile çarpma!

$v_1' = K . e^t$

$v_1'' = K . e^t$

$$\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 4y = e^t$$

$K . e^t - 4 . K . e^t + 4 . e^t = e^t$

$K = 1$

$\boxed{v_1 = e^t}$

$v_2 = z . e^{2t}$

$v_2' = z' e^{2t} + z . 2 e^{2t}$

$v_2'' = z'' e^{2t} + 4z' e^{2t} + z . 4 . e^{2t}$

$$\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 4y = t . e^{2t}$$

$$z'' e^{2t} + \cancel{4z' e^{2t}} + \cancel{z 4 e^{2t}} - \cancel{4z' e^{2t}} - \cancel{4z . 2 . e^{2t}} + \cancel{4z e^{2t}} = t . e^{2t}$$

$$z'' e^{2t} = t . e^{2t}$$

Bu yeni denk. kar. denk. kök. 0 var. mı? $r^2 = 0$ $r_1 = 0$ $r_2 = 0$ 2 kez var. 2 kez t ile çarp.

$z'' = t$

$z_2 = t^2 (at + b) = at^3 + bt^2$

GIPTA

$z'' = t$

$\underbrace{6a}_{1}t + \underbrace{2b}_{0} = t \qquad a = 1/6 \quad b = 0$

$z_2 = at^3 + bt^2$ idi

$z_2 = 1/6\, t^3$

$V_2 = z \cdot e^{2t}$ idi

$V_2 = \left(\frac{1}{6} t^3 e^{2t}\right)$

$y = u + v$

$y = u + v_1 + v_2 = (c_1 + c_2 t) \cdot e^{2t} + e^t + \left(\frac{1}{6} t^3 e^{2t}\right)$

$x = e^t$ idi $t = \ln x$

$y = (c_1 + c_2 \ln x) \cdot x^2 + x + \left(\frac{1}{6} (\ln x)^3 \cdot x^2\right)$

Örn. $x^3 y''' - 2x^2 y'' = 2x^3 - x$

$x = e^t \qquad D = \frac{d}{dt}$

$y' = e^{-t} \cdot Dy \qquad = e^{-t} \frac{dy}{dt}$

$y'' = \underbrace{e^{-2t} \cdot D(D-1)y}_{D^2y - Dy} \qquad = e^{-2t}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)$

$y''' = e^{-3t} \cdot \underbrace{D(D-1)(D-2)y}_{(D^2-D)(D-2)y} \qquad = e^{-3t}\left(\frac{d^3y}{dt^3} - \frac{3d^2y}{dt^2} + \frac{2dy}{dt}\right)$

$(D^3 - 2D^2 - D^2 + 2D)y$

$(D^3 - 3D^2 + 2D)y$

$\cancel{e^{3t}} \cdot \cancel{e^{-3t}} \cdot \left(\frac{d^3y}{dt^3} - \frac{3d^2y}{dt^2} + \frac{2dy}{dt}\right) - 2 \cdot \cancel{e^{2t}} \cdot \cancel{e^{-2t}} \cdot \left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right) = 2e^{3t} - e^t$

$\frac{d^3y}{dt^3} - \frac{5d^2y}{dt^2} + \frac{4dy}{dt} = 2e^{3t} - e^t$

$r^3 - 5r^2 + 4r = 0 \qquad r_1 = 0$

$r(r^2 - 5r + 4) = 0 \qquad r_2 = 1$

$\qquad -4 \quad -1 \qquad r_3 = 4$

$u = c_1 \cdot e^{0t} + c_2 \cdot e^{1t} + c_3 e^{4t}$

$V_1 = K \cdot e^{3t}$ (Kar. denk. 3 yok t ile çarpma)

$V_1' = 3K \cdot e^{3t}$

$V_1'' = 9K \cdot e^{3t}, \quad V_1''' = 27K \cdot e^{3t}$

$\frac{d^3y}{dt^3} - \frac{5d^2y}{dt^2} + \frac{4dy}{dt} = 2e^{3t}$

$27K \cdot e^{3t} - 45Ke^{3t} + 12Ke^{3t} = 2 \cdot e^{3t}$

$-6K = 2$

$K = -1/3 \qquad V_1 = -\frac{1}{3} e^{3t}$

$V_2 = Kte^t$ (K.D. K. 1 adet 1 var t ile çarp)

$V_2' = Ke^t + Kte^t$

$V_2'' = Ke^t + Ke^t + Kte^t$

$V_2''' = Ke^t + Ke^t + Ke^t + Kte^t$

$\frac{d^3y}{dt^3} - \frac{5d^2y}{dt^2} + \frac{4dy}{dt} = -e^t$

$3Ke^t + \cancel{Kte^t} - 10Ke^t - \cancel{5Kte^t} + 4Ke^t + \cancel{4Kte^t} = -e^t$

$-3Ke^t = -e^t$

$K = \frac{1}{3} \qquad \boxed{V_2 = \frac{1}{3} te^t}$

$y = u + v_1 + v_2$

$y = c_1 + c_2 e^t + c_3 e^{4t} - \frac{1}{3} e^{3t} + \frac{1}{3} te^t$

$x = e^t \qquad t = \ln x$

$y = c_1 + c_2 x + c_3 x^4 - \frac{1}{3} x^3 + \frac{1}{3} (\ln x) x$

GIPTA

Ödev: $x^2y'' + xy' + 4y = x^2 \sin(2\ln x)$

$x = e^t \quad D = \frac{d}{dt}$

$y' = e^{-t} Dy \rightarrow e^{-t}\left(\frac{dy}{dt}\right)$

$y'' = e^{-2t} D(D-1)y \rightarrow e^{-2t}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)$

$\cancel{e^{2t}} \cdot \cancel{e^{-2t}}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right) + \cancel{e^t} \cdot \cancel{e^{-t}} \cdot \frac{dy}{dt} + 4y = e^{2t} \cdot \sin(2\ln e^t)$

$\frac{d^2y}{dt^2} + 4y = e^{2t} \cdot \sin(2t)$

$r^2 + 4 = 0$, $r_{1,2} = \mp 2i$ $\Big\}$ $u = e^{0t}(c_1 \sin 2t + c_2 \cos 2t)$

$u = c_1 \sin 2t + c_2 \cos 2t$

$v = z \cdot e^{2t}$

$v' = z' e^{2t} + z \cdot 2 \cdot e^{2t}$

$v'' = z'' e^{2t} + z' \cdot 2 \cdot e^{2t} + 2z' e^{2t} + 4ze^{2t}$

$z'' e^{2t} + 4z' e^{2t} + 4ze^{2t} + 4ze^{2t} = e^{2t} \cdot \sin(2t)$

$z'' + 4z' + 8z = \sin(2t)$

$r^2 + 4r + 8 = 0$

$r_{1,2} = \frac{-4 \mp \sqrt{16 - 4 \cdot 8}}{2} = -2 \mp 2i$

$z_2 = (a \sin 2t + b \cos 2t)t$

$z_2' = (2a \cos 2t - 2b \sin 2t)t + (a \sin 2t + b \cos 2t)$

$z_2'' = (-4a \sin 2t - 4b \cos 2t)t + (2a \cos 2t - 2b \sin 2t) + 2a \cos 2t - 2b \sin 2t$

İkinci Mertebeden Yüksek Dereceden
Bazı Özel Durumlu Diferansiyel Denklemler $f(x, y, y', y'') = 0$

1°) y'yi içermeyen dif. denk. $f(x, y', y'') = 0$

$\frac{dy}{dx} = y' = p \qquad y'' = \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{dp}{dx}$

★ $x \cdot \frac{d^2y}{dx^2} - 2 \cdot \frac{dy}{dx} = 12x^3$

$\frac{dy}{dx} = p \qquad \frac{d^2y}{dx^2} = \frac{dp}{dx}$

$x \cdot \frac{dp}{dx} - 2 \cdot p = 12x^3$

$\frac{dp}{dx} - \frac{2}{x} p = 12x^2$ LDD

$P(x) = -\frac{2}{x}$

$Q(x) = 12x^2$

$\mu = e^{\int P(x)dx}$

$\mu = e^{\int -\frac{2}{x}dx}$

$\mu = e^{-2\ln x} = e^{\ln x^{-2}}$

$\mu = x^{-2}$

$\mu = \frac{1}{x^2}$

$p\cancel{y} = \frac{1}{\mu}\int \mu Q dx$

$p = x^2\left(\int \frac{1}{x^2} 12x^2 dx\right)$

$p = x^2(12x + c_1)$

$\frac{dy}{dx} = x^2(12x + c_1)$

$\int dy = \int x^2(12x + c_1)dx$

$\boxed{y = \frac{12x^4}{4} + \frac{c_1 x^3}{3} + c_2}$

2°) x' i içermeyen dif. denk $f(y, y', y'') = 0$

$y' = p$

$$y'' = \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{dp}{dx}\cdot\frac{dy}{dy} = \frac{dp}{dy}\cdot\underbrace{\frac{dy}{dx}}_{p} = p\cdot\frac{dp}{dy}$$

★ $y\cdot y'' = y^2\cdot y' + (y')^2$

$y\cdot p\cdot\frac{dp}{dy} = y^2\cdot p + p^2$

$p\left(y\frac{dp}{dy} - y^2 - p\right) = 0$

$p = 0 \rightsquigarrow \frac{dy}{dx} = 0 \Rightarrow y = k$

$y\frac{dp}{dy} - y^2 - p = 0$

$\frac{dp}{dy} - \frac{p}{y} = y$, LDD

$P(y) = -\frac{1}{y}$

$Q(y) = y$

$\mu = e^{\int -\frac{1}{y}dy}$

$\mu = \frac{1}{y}$

$p = y\left[\int \frac{1}{y}\cdot y\cdot dy\right]$

$p = y(y + c_1)$

$\frac{dy}{dx} = y(y + c_1)$

$\int \frac{dy}{y(y+c_1)} = \int dx$

$\frac{A}{y} + \frac{B}{y+c_1} = \frac{1}{y(y+c_1)}$

$(y+c_1) \quad y$

$A(y + c_1) + By = 1$

$A + B = 0 \qquad B = -A$

$Ac_1 = 1 \qquad A = \frac{1}{c_1} \qquad B = -\frac{1}{c_1}$

$\int \frac{1}{c_1 y}dy + \int \frac{-1}{c_1(y+c_1)}dy = \int dx$

$\frac{1}{c_1}\ln y - \frac{1}{c_1}\ln|y + c_1| = x + c_2$

★ $y''' - 4y' = (x^2+4) + e^{2x}$

$r^3 - 4r = 0$

$r(r^2-4) = 0 \qquad r_1 = 0,\ r_2 = 2,\ r_3 = -2$

$u = c_1e^{0x} + c_2e^{2x} + c_3e^{-2x}$

$u = c_1 + c_2e^{2x} + c_3e^{-2x}$

$v_1 = (ax^2+bx+c)\cdot x$ (kar. denk. kök. 1 tane over. x ile çarp)

$v_1 = ax^3 + bx^2 + cx$

$v_1' = 3ax^2 + 2bx + c$

$v_1'' = 6ax + 2b$

$v_1''' = 6a$

$6a - 12ax^2 - 8bx - 4c = x^2 + 4$

$a = -1/12 \quad b = 0 \quad c = -9/8$

$$\boxed{v_1 = -\frac{1}{12}x^3 - \frac{9}{8}x}$$

$v_2 = k\cdot x\cdot e^{2x}$ (1 adet 2 var 1 kez x ile çarp)

$v_2' = ke^{2x} + 2kxe^{2x}$

$v_2'' = 2ke^{2x} + 2ke^{2x} + 4kxe^{2x}$

$v_2''' = 8ke^{2x} + 4ke^{2x} + 8kxe^{2x}$

$(12k + \cancel{8kx} - 4k - \cancel{8kx})\cdot e^{2x} = e^{2x}$

$8k = 1 \qquad k = \frac{1}{8}$

$v_2 = \frac{1}{8}xe^{2x}$

$y = u + v = u + v_1 + v_2$

$y = \ldots$

1. $y''-2y'+y=\frac{xe^x}{1+x^2}$

$r^2-2r+1=0$

$(r-1)^2=0$

$r_1=r_2=1$

$y=(c_1+c_2x)e^x$

$y=c_1e^x+c_2xe^x$

$$\boxed{\begin{aligned} c_1'e^x+c_2'xe^x &= 0 \\ c_1'e^x+c_2'(e^x+xe^x) &= \frac{xe^x}{1+x^2} \end{aligned}}$$

$c_2'=\frac{x}{1+x^2} \Rightarrow \frac{dc_2}{dx}=\frac{x}{1+x^2} \Rightarrow \int dc_2=\int\frac{x}{1+x^2}dx \Rightarrow c_2=\frac{1}{2}\ln(1+x^2)+k_2$

$c_1'=\frac{-x^2}{1+x^2} \Rightarrow \frac{dc_1}{dx}=-1+\frac{x^2}{1+x^2} \Rightarrow \int dc_1=\int\left(\frac{1}{1+x^2}-1\right)dx$

$c_1=(\text{Arctan}x-x)+k_1$

$y=c_1e^x+c_2xe^x$

İkinci Mertebeden Lineer Denklemlerin Seri Çözümleri

$P(x).y''+Q(x).y'+R(x).y=0$ şeklinde dif. denklemlerin seri ile çözümünü inceleyelim.

Örnek denk;

$a(x-x_0)^2y''+b(v-x_0)y'+cy=0$ Cauchy-Euler Dif. Denk

$x^2y''+xy'+(x^2-\lambda^2)y=0$ Bessel Denklemi

$(1-x^2)y''-2xy'+xy=0$ Legendre Dif. Denklemi

$y''-\lambda xy=0$ Airy Dif. Denk.

$y''-2xy'+2ny=0$ Hermite " "

$(1-x^2)y''-xy'+n^2y=0$ Chebyshev Denk.

Teorem: Eğer $x=x_0$ noktası

$$y''+\frac{Q(x)}{P(x)}y'+\frac{R(x)}{P(x)}y=0$$

dif. denkleminin bir adi noktası ise bu noktanın uygun bir civarında

$y=a_0+a_1(x-x_0)+a_2(x-x_0)^2+\dots=\sum_{n=0}^{\infty}a_n(x-x_0)^n$ şek. bir seri çözümü bulunur.

Lemma: Eğer $f(x)=\sum_{n=0}^{\infty}a_n(x-x_0)^n$ kuvvet serisi bir açık (a,b) aralığında yakınsak ise bu aralıkta $f(x)$'in türevleri;

$$f'(x)=\sum_{n=0}^{\infty}a_n.n.(x-x_0)^{n-1}$$

$$f''(x)=\sum_{n=0}^{\infty}a_n.n.(n-1).(x-x_0)^{n-2}$$

Tanım: Eğer $\forall x$ için;

1) $\sum_{n=0}^{\infty}a_n(x-x_0)^n=\sum_{n=0}^{\infty}b_n(x-x_0)^n$ $a_0=b_0$, $a_1=b_1$, $\vdots$, $a_n=b_n$

2) $\sum_{n=0}^{\infty}a_n(x-x_0)^n=0 \rightarrow a_0=a_1=\dots=a_n=0$

3) $\sum_{n=0}^{\infty}\frac{x^n}{n!.3^n}=\sum_{k=0}^{\infty}\frac{x^k}{k!.3^k}$

☆ $\sum_{n=0}^{\infty}\frac{x^n}{n!.3^n}=\sum_{n=0+5}^{\infty}\frac{x^{n-5}}{n-5!.3^{n-5}}$

$\sum_{n=0-7}^{\infty}\frac{x^{n+7}}{n+7!.3^{n+7}}$

Soru: $y'' - (x-2)y' + 2y = 0$ dif. denk. $x=2$ nokt. civarındaki seri çözümünü bul. (veya iki farklı lin. bağımsız çözümünü bul.)

1: 0Σ 2: Σ $(x-x_0)$ en küçüğe eşitle

3- Σ eşitle (en büyüğe göre)

$$y = \sum_{n=0}^{\infty} a_n (x-2)^n$$

$$y' = \sum_{n=0}^{\infty} a_n . n . (x-2)^{n-1}$$

$$y'' = \sum_{n=0}^{\infty} a_n . n . (n-1) . (x-2)^{n-2}$$

$$\sum_{n=0}^{\infty} a_n . n(n-1)(x-2)^{n-2} - (x-2)\sum_{n=0}^{\infty} a_n . n (x-2)^{n-1} + 2\sum_{n=0}^{\infty} a_n (x-2)^n = 0$$

$$\sum_{n=0}^{\infty} a_n . n(n-1)(x-2)^{n-2} - \sum_{n=0+2}^{\infty} a_{n-2}(n-2)(x-2)^{n-2} + \sum_{n=0+2}^{\infty} 2a_{n-2}(x-2)^{n-2} = 0$$

$$\underbrace{\sum_{n=0}^{\infty} a_n . n(n-1)(x-2)^{n-2}} - \sum_{n=2}^{\infty} a_{n-2} . (n-2)(x-2)^{n-2} + \sum_{n=2}^{\infty} 2a_{n-2}(x-2)^{n-2} = 0$$

1-) $n=0 \rightarrow a_0 . 0 . (0-1)(x-2)^{-2} = 0$

2-) $n=1 \rightarrow a_1 . 1 . (1-1)(x-2)^{-1} = 0$

$\} \ \cancel{I}^{0} + \cancel{II}^{0} + \sum_{n=2}^{\infty} \dots$

$$\sum_{n=2}^{\infty} a_n . n(n-1)(x-2)^{n-2} - \sum_{n=2}^{\infty} a_{n-2}(n-2)(x-2)^{n-2} + \sum_{n=2}^{\infty} 2a_{n-2}(x-2)^{n-2} = 0$$

$$\sum_{n=2}^{\infty} \Big[\underbrace{a_n . n(n-1) - (n-2)a_{n-2} + 2a_{n-2}}_{0} \Big] (x-2)^{n-2} = 0$$

$$a_n = \frac{[(n-2)-2]\,a_{n-2}}{n(n-1)}, \quad n \geq 2$$

$$a_n = \frac{(n-4)}{n(n-1)} . a_{n-2} \qquad n \geq 2$$

$n=2 \Rightarrow a_2 = \frac{-2}{2} . a_0$ $\boxed{a_2 = -a_0}$

$n=3 \Rightarrow a_3 = \frac{-1}{3.(2)} . a_1$ $\boxed{a_3 = -\frac{1}{6} . a_1}$

$n=4 \Rightarrow a_4 = 0 . a_2$ $\boxed{a_4 = 0}$

$n=5 \Rightarrow a_5 = \frac{1}{5.4} . a_3$ $\boxed{= \frac{-1}{120} a_1 = a_5}$

$$y = \sum_{n=0}^{\infty} a_n (x-2)^n = a_0 + a_1(x-2) + a_2(x-2)^2 + a_3(x-2)^3 + a_4(x-2)^4 + a_5(x-2)^5 + \dots$$

$$y = a_0 + a_1(x-2) - a_0(x-2)^2 - \frac{1}{6} a_1 (x-2)^3 - \frac{1}{120} a_1 (x-2)^5 + \dots$$

$$y = a_0\left(1 - (x-2)^2 + \dots\right) + a_1\left((x-2) - \frac{1}{6}(x-2)^3 - \frac{1}{120}(x-2)^5 + \dots\right)$$

4) $y''+xy'+y=0$ dif. denk. $x=1$ nokt. civarındaki seri çözümünü bul.

$$y=\sum_{n=0}^{\infty} a_n (x-1)^n$$

$$y'=\sum_{n=0}^{\infty} a_n \cdot n (x-1)^{n-1}$$

$$y''=\sum_{n=0}^{\infty} a_n \cdot n (n-1)(x-1)^{n-2}$$

$$\sum_{n=0}^{\infty} a_n \cdot n(n-1)(x-1)^{n-2} + [(x-1)+1]\sum_{n=0}^{\infty} a_n \cdot n (x-1)^{n-1} + \sum_{n=0}^{\infty} a_n (x-1)^n = 0$$

$$\sum_{n=0}^{\infty} a_n \cdot n(n-1)(x-1)^{n-2} + (x-1)\sum_{n=0}^{\infty} a_n \cdot n (x-1)^{n-1} + \sum_{n=0}^{\infty} a_n \cdot n (x-1)^{n-1} + \sum_{n=0}^{\infty} a_n (x-1)^n = 0$$

$$\sum_{n=0}^{\infty} a_n \cdot n(n-1)(x-1)^{n-2} + \sum_{n=0+2}^{\infty} a_{n-2} \cdot (n-2)(x-1)^{n-2} + \sum_{n=0+1}^{\infty} a_{n-1} \cdot (n-1)(x-1)^{n-1-1} + \sum_{n=0+2}^{\infty} a_{n-2} (x-1)^{n-2} = 0$$

$$\sum_{n=0}^{\infty} a_n \cdot n(n-1)(x-1)^{n-2} + \sum_{n=2}^{\infty} a_{n-2} \cdot (n-2)(x-1)^{n-2} + \sum_{n=1}^{\infty} a_{n-1} (n-1)(x-1)^{n-2} + \sum_{n=2}^{\infty} a_{n-2} (x-1)^{n-2} = 0$$

I $n=0 \to a_0 \cdot 0 \cdot -1 \ldots = 0$ | ✓ | $n=1 \to \quad = 0$ | ✓

II $n=1 \to \quad = 0$

$$\sum_{n=2}^{\infty} \underbrace{\left[a_n \cdot n(n-1) + a_{n-2} \cdot (n-2) + a_{n-1}(n-1) + a_{n-2} \right]}_{0} (x-1)^{n-2} = 0$$

$$a_n = -\frac{a_{n-2} \cdot \cancel{(n-2+1)} + a_{n-1} \cancel{(n-1)}}{n \cancel{(n-1)}} \qquad \boxed{a_n = -\frac{a_{n-2}+a_{n-1}}{n} \quad n \geq 2}$$

$n=2 \to \boxed{a_2 = -\frac{a_0}{2} - \frac{a_1}{2}}$

$n=3 \to a_3 = -\frac{a_1}{3} - \frac{a_2}{3} = -\frac{a_1}{3} - \frac{1}{3}\left(-\frac{a_0}{2} - \frac{a_1}{2}\right) \quad \boxed{a_3 = \frac{a_0}{6} - \frac{a_1}{6}}$

$n=4 \to a_4 = -\frac{a_2}{4} - \frac{a_3}{4} = -\frac{1}{4}\left(\frac{a_0}{6} - \frac{a_1}{6}\right) - \frac{1}{4}\left(-\frac{a_0}{2} - \frac{a_1}{2}\right)$

$$\boxed{a_4 = \frac{a_0}{12} + \frac{a_1}{6}}$$

$$y = \sum_{n=0}^{\infty} a_n (x-1)^n = a_0 + a_1(x-1) + a_2(x-1)^2 + a_3(x-1)^3 + a_4(x-1)^4 + a_5(x-1)^5 + \ldots$$

$$y = a_0 + a_1(x-1) + \left(-\frac{a_0}{2} - \frac{a_1}{2}\right)(x-1)^2 + \left(\frac{a_0}{6} - \frac{a_1}{6}\right)(x-1)^3 + \left(\frac{a_0}{12} + \frac{a_1}{6}\right)(x-1)^4 + \ldots$$

$$y = a_0\left(1 - \frac{1}{2}(x-1)^2 + \frac{1}{6}(x-1)^3 + \frac{1}{12}(x-1)^4 + \ldots\right) + a_1\left((x-1) - \frac{1}{2}(x-1)^2 - \frac{1}{6}(x-1)^3 + \frac{1}{6}(x-1)^4 + \ldots\right)$$

$x=-2$ ndct.

$$y''-(x+2)y'=0$$

$$y=\sum_{n=0}^{\infty} a_n(x-(-2))^n$$

$$y'=\sum_{n=0}^{\infty} a_n \cdot n(x+2)^{n-1}$$

$$y''=\sum_{n=0}^{\infty} a_n \cdot n(n-1)(x+2)^{n-2}$$

$$\sum_{n=0}^{\infty} a_n \cdot n(n-1)(x+2)^{n-2} - (x+2)\sum_{n=0}^{\infty} a_n \cdot n(x+2)^{n-1} = 0$$

$$\sum_{n=0}^{\infty} a_n \cdot n(n-1)(x+2)^{n-2} - \sum_{n=0+2}^{\infty} a_{n-2} \cdot (n-2)(x+2)^{n-2} = 0$$

$I \to n=0 \to 0$
$II \to n=1 \to 0$

$$\underbrace{I+II}_{0} + \sum_{n=2}^{\infty} a_n \cdot n(n-1)(x+2)^{n-2} - \sum_{n=2}^{\infty} a_{n-2} (n-2)(x+2)^{n-2} = 0$$

$$\sum_{n=2}^{\infty}\left[\underbrace{a_n \cdot n(n-1) - a_{n-2}(n-2)}_{0}\right](x+2)^{n-2} = 0$$

$$\boxed{a_n = \frac{a_{n-2}(n-2)}{n(n-1)} \quad n \geq 2}$$

$n=2 \to a_2 = 0$

$n=3 \to a_3 = \frac{a_1}{3 \cdot 2} = \frac{a_1}{6}$

$n=4 \to a_4 = \frac{a_2 \cdot 2}{4 \cdot 3} = 0$

$n=5 \to a_5 = \frac{3a_3}{5 \cdot 4} = \frac{3a_1}{5!}$

$$y=\sum_{n=0}^{\infty} a_n(x+2)^n = a_0 + a_1(x+2) + a_2(x+2)^2 + a_3(x+2)^3 + a_4(x+2)^4 + a_5(x+2)^5 + \dots$$

$$y = a_0 + a_1(x+2) + \frac{a_1}{6}\cdot(x+2)^3 + \frac{3a_1}{5!}(x+2)^5 + \dots$$

Laplace Dönüşümü

Tanım: $K(x,t) = \begin{cases} 0 & ; \quad t<0 \\ e^{-st} & \quad t \geq 0 \end{cases}$ çekirdek fonk. olmak üzere

$$\int_{-\infty}^{+\infty} K(x,t) \cdot f(t) \cdot dt = \cancel{\int_{-\infty}^{0} K(x,t) \cdot f(t) \cdot dt} + \int_{0}^{\infty} K(x,t) \cdot f(t) \cdot dt = \int_0^{\infty} e^{-st} \cdot f(t) \cdot dt = \mathcal{L}\{f(t)\}$$

($f(t)$ fonk. Laplace dönüşümü denir.)