**4.)** $\left.\begin{array}{l} 2\dfrac{dx}{dt}+\dfrac{dy}{dt}+x+5y=4t \\ \dfrac{dx}{dt}+\dfrac{dy}{dt}+2x+2y=2 \end{array}\right\}$ diferansiyel denklem sisteminin genel çözümünü **determinant** yöntemini kullanarak çözünüz.

| **YTÜ - Fen-Edebiyat Fakültesi** **Final Sınav Soru ve Cevap Kâğıdı** | | | | **NOT TABLOSU** | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | 1.S | 2.S | 3.S | 4.S | TOPLAM |
| Adı Soyadı | | | | | | | | |
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| Dersi veren Öğretim Üyesinin Adı Soyadı | | | | | İmza | | | |

YÖK nun 2547 sayılı Kanunun *Öğrenci Disiplin Yönetmeliğinin* 9. Maddesi olan ***"Sınavlarda kopya yapmak ve yaptırmak veya buna teşebbüs etmek"*** fiili işleyenler bir veya iki yarıyıl uzaklaştırma cezası alırlar.

**1.)** $\dfrac{d^2y}{dt^2}+2\dfrac{dy}{dt}+y=t.e^{-t}$ diferansiyel denkleminin $y(0)=1$, $y'(0)=2$ başlangıç koşullarına uyan çözümünü, Laplace dönüşümünü kullanarak bulunuz.

2.) $y'' + xy' + y = 0$ , $y(0) = 2$ $y'(0) = 3$ başlangıç değer probleminin kuvvet seri çözümünü bulunuz

3.) $\left(2\sqrt{xy} - x\right)dy + ydx = 0$ diferansiyel denkleminin genel çözümünü bulunuz.

1) $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = t \cdot e^{-t}$ $\quad y(0)=1,\ y'(0)=2$

$L\{y''(t)\} + 2L\{y'(t)\} + L\{y(t)\} = L\{t.e^{-t}\}$

$s^2Y(s) - s\underbrace{y(0)}_{1} - \underbrace{y'(0)}_{2} + 2[sY(s) - \underbrace{y(0)}_{1}] + Y(s) = \frac{1}{(s+1)^2}$

$(s^2+2s+1)\,Y(s) = s+4+\frac{1}{(s+1)^2}$

B

=0 ixe

$Y(s) = \frac{s+1-1}{(s+1)^2} + \frac{4}{(s+1)^2} + \frac{1}{(s+1)^4}$

$Y(s) = \frac{s^3+6s^2+9s+5}{(s+1)^4}$

$= \frac{1}{s+1} - \frac{1}{(s+1)^2} + \frac{4}{(s+1)^2} + \frac{1}{(s+1)^4}$

$L^{-1}[Y(s)] = L^{-1}\left[\frac{1}{s+1}\right] + L^{-1}\left[\frac{3}{(s+1)^2}\right] + L^{-1}\left[\frac{1}{(s+1)^4}\right]$

$y(t) = e^{-t} + 3t.e^{-t} + \frac{e^{-t}.t^3}{3!}$

A, B, C, D getirdi. Sadece A doğru ve çözdü

$Y(s) = \frac{(s+4)(s+1)^2+1}{(s+1)^4} = \frac{(s+4)(s^2+2s+1)+1}{(s+1)^4} =$

$\frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{(s+1)^3} + \frac{D}{(s+1)^4}$ $= \frac{s^3+2s^2+s+4s^2+8s+4+1}{(s+1)^4}$

$A=1 \quad B=3 \quad C=0 \quad D=1$

$= \frac{s^3+6s^2+9s+5}{(s+1)^4}$

① $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = te^{-t}$ $\quad y(0)=1 \; ; \; y'(0)=2$

$\mathcal{L}\{y''\} + 2\mathcal{L}\{y'\} + \mathcal{L}\{y\} = \mathcal{L}\{te^{-t}\}$

$s^2Y(s) - s\underbrace{y(0)}_{1} - \underbrace{y'(0)}_{2} + 2sY(s) - 2\underbrace{y(0)}_{1} + Y(s) = \frac{1}{(s+1)^2}$

$(s^2+2s+1)Y(s) - s - 4 = \frac{1}{(s+1)^2}$

$$\boxed{Y(s) = \frac{s+4}{(s+1)^2} + \frac{1}{(s+1)^4}}$$

1\. yol $\quad Y(s) = \frac{s+1-1}{(s+1)^2} + \frac{4}{(s+1)^2} + \frac{1}{(s+1)^4}$ ③

$Y(s) = \frac{1}{(s+1)} - \frac{1}{(s+1)^2} + \frac{4}{(s+1)^2} + \frac{1}{(s+1)^4}$

$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{(s+1)}\right\} - \underbrace{\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2}\right\} + 4\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2}\right\}}_{3\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2}\right\}} + \mathcal{L}^{-1}\left\{\frac{1}{(s+1)^4}\right\}$

$$\boxed{y(t) = e^{-t} + 3te^{-t} + \frac{e^{-t}t^3}{3!}}$$ //

2\. yol $\quad Y(s) = \frac{s+1}{(s+1)^2} + \frac{3}{(s+1)^2} + \frac{1}{(s+1)^4}$

3\. yol $\quad \frac{s+4}{(s+1)^2} = \frac{A}{s+1}_{(s+1)} + \frac{B}{(s+1)^2}$

$As + A + B = s + 4$

$A = 1 \quad A + B = 4 \Rightarrow B = 3$

4\. yol $\quad Y(s) = \frac{(s+4)(s+1)^2 + 1}{(s+1)^4} = \frac{s^3 + 6s^2 + 9s + 5}{(s+1)^4} = \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{(s+1)^3} + \frac{D}{(s+1)^4}$

$A=1 \quad B=3 \quad C=0 \quad D=1$

(2) $y'' + xy' + y = 0 \qquad y(0)=2, \quad y'(0)=3$ seri açılım

$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$y' = \sum_{n=1}^{\infty} a_n . n . x^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} a_n . n . (n-1) x^{n-2}$$

$$\sum_{n=2}^{\infty} a_n . n(n-1) x^{n-2} + \sum_{n=1}^{\infty} a_n . n . x^n + \sum_{n=0}^{\infty} a_n . x^n = 0$$

$$\sum_{n=0}^{\infty} a_{n+2} (n+2)(n+1) x^n + \sum_{n=1}^{\infty} a_n . n . x^n + \sum_{n=0}^{\infty} a_n . x^n = 0$$

$$\sum_{n=0}^{\infty} a_{n+2} (n+2)(n+1) x^n + \sum_{n=0}^{\infty} a_n . n . x^n + \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=0}^{\infty} \left[ a_{n+2} (n+2)(n+1) + a_n (n+1) \right] x^n = 0$$

$$a_{n+2} = -\frac{1}{(n+2)} a_n \qquad n = 0, 1, 2 \ldots$$

$n=0 \qquad a_2 = -\frac{1}{2} a_0$

$n=1 \qquad a_3 = -\frac{1}{3} a_1$

$n=2 \qquad a_4 = -\frac{1}{4} a_2 = \left(-\frac{1}{4}\right)\left(-\frac{1}{2}\right) a_0 = \frac{1}{2.4} a_0$

$n=3 \qquad a_5 = -\frac{1}{5} a_3 = \left(-\frac{1}{5}\right)\left(-\frac{1}{3}\right) a_1 = \frac{1}{3.5} a_1$

2.deva

$$y_{GU} = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$

$$= a_0\left[1 - \frac{1}{2}x^2 + \frac{1}{2.4}x^4 - + \cdots\right]$$

$$+ a_1\left[x - \frac{1}{3}x^3 + \frac{1}{3.5}x^5 - + \cdots\right]$$

$$y(0) = 2 \Rightarrow a_0 = 2$$

$$y' = a_0\left[-x + \frac{4}{2.4}x^3 + \cdots + \frac{(-1)^n x^{2n}}{2.4\ldots(2n)} + \cdots\right]$$

$$+ a_1\left[1 - x^2 + \frac{5}{3.5}x^3 - + \cdots + \frac{(-1)^n x^{2n+1}}{3.5\ldots(2n+1)} + \cdots\right]$$

$$y'(0) = 3 \Rightarrow a_1 = 3$$

$$y_{öa} = 2\left[1 - \frac{1}{2}x^2 + \frac{1}{2.4}x^4 - + \cdots\right]$$

$$+ 3\left[x - \frac{1}{3}x^3 + \frac{1}{3.5}x^5 - + \cdots\right]$$

$$y = 2 + 3x - x^2 - x^3 + \frac{1}{4}x^4 + \frac{1}{5}x^5 \cdots$$

3.) $\left(2\sqrt{xy}-x\right)dy+y\,dx=0$ d.d-cöz.

$\frac{1}{x}/ \left(2\sqrt{xy}-x\right)dy+y\,dx=0$

$\left(2\sqrt{\frac{y}{x}}-1\right)dy+\frac{y}{x}dx=0$ $\qquad (2\sqrt{u}-1)\left(\cancel{4}+x\frac{du}{dx}\right)+u=0$

$\frac{y}{x}=u \qquad \frac{dy}{dx}=u\frac{dx}{dx}+x\frac{du}{dx}$

$(2\sqrt{u}-1)(u\,dx+x\,du)+u\,dx=0$

$(2\sqrt{u}-1)u\,dx+(2\sqrt{u}-1)x\,du+u\,dx=0$

$\left[2u\sqrt{u}-\cancel{u}+\cancel{u}\right].dx+(2\sqrt{u}-1)x\,du=0$

$\int\frac{dx}{x}+\int\frac{2\sqrt{u}-1}{2u\sqrt{u}}du=0$

$\frac{dy}{dx}=\frac{-y/x}{2\sqrt{\frac{y}{x}}-1}$ $\qquad \int\frac{dx}{x}+\int\frac{du}{u}-\frac{1}{2}\int\frac{du}{u^{3/2}}=0$

$\frac{-u}{2\sqrt{u}-1}=u\cancel{+}x\frac{du}{dx}$ $\qquad \ln|x|+\ln|u|-\frac{1}{2}\frac{u^{-\frac{1}{2}}}{-\frac{1}{2}}+\ln c=0$

$\ln|x|+\ln\left|\frac{y}{x}\right|+\sqrt{\frac{x}{y}}+\ln c=0$

$\ln\left|x.\frac{y}{x}.c\right|=-\sqrt{\frac{x}{y}}$

$\frac{dx}{x}=$

$+\int\frac{2\sqrt{u}-1}{2u\sqrt{u}}=0$ $\qquad y\cdot c=e^{-\sqrt{\frac{x}{y}}}$

$y.e^{\sqrt{\frac{x}{y}}}=K$

4. $2\dfrac{dx}{dt} + \dfrac{dy}{dt} + x + 5y = 4t$ d.d.s determinant yönt. ile çöz.

$\dfrac{dx}{dt} + \dfrac{dy}{dt} + 2x + 2y = 2$

$$\left.\begin{aligned}(2D+1)x + (D+5)y &= 4t\\ (D+2)x + (D+2)y &= 2\end{aligned}\right\}$$

$$\Delta = \begin{vmatrix} 2D+1 & D+5 \\ D+2 & D+2 \end{vmatrix} = 2D^2+4D+D+2-D^2-7D-10$$

$$\Delta = D^2-2D-8$$

$$x = \frac{\Delta_1}{\Delta} = \frac{1}{D^2-2D-8}\begin{vmatrix} 4t & D+5 \\ 2 & D+2 \end{vmatrix} = \frac{1}{D^2-2D-8}(4+8t-10)$$

$$(D^2-2D-8)\,x = 8t-6$$

$$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} - 8x = 8t-6$$

$$r^2-2r-8 = (r+2)(r-4) = 0 \qquad r_1=-2 \quad r_2=4$$

$$x_h = c_1 e^{-2t} + c_2 e^{4t}$$

$$\left.\begin{aligned} x_ö &= At+B \\ x' &= A \\ x'' &= 0 \end{aligned}\right\} \quad -2A-8(At+B) = 8t-6$$

$$\left.\begin{aligned} -8A &= 8 & A &= -1 \\ -2A-8B &= -6 & B &= 1 \end{aligned}\right\} x_ö = -t+1$$

$$x_{GÇ} = c_1 e^{-2t} + c_2 e^{4t} - t + 1$$

1. yol: $2\frac{dx}{dt} + \frac{dy}{dt} + x + 5y = 4t$

$-1/ \quad \frac{dx}{dt} + \frac{dy}{dt} + 2x + 2y = 2$

$+$

$$\boxed{\frac{dx}{dt} - x + 3y = 4t - 2}$$

$-1/ \quad x = c_1 e^{-2t} + c_2 e^{4t} - t + 1$

$$\frac{dx}{dt} = -2c_1 e^{-2t} + 4c_2 e^{4t} - 1$$

$$-3c_1 e^{-2t} + 3c_2 e^{4t} + t - 2 + 3y = 4t - 2$$

$$3y_{Gu} = 3c_1 e^{-2t} - 3c_2 e^{4t} + 3t$$

$$y_{Gu} = c_1 e^{-2t} - c_2 e^{4t} + t$$

2.yol: $y = \frac{\Delta_2}{\Delta} = \frac{1}{D^2-2D-8}\begin{vmatrix} 2D+1 & 4t \\ D+2 & 2 \end{vmatrix}$

$(D^2-2D-8)y = 2-4-8t$

$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} - 8y = -8t-2$

$r^2-2r-8=(r+2)(r-4)=0 \quad r_1=-2 \quad r_2=4$

$y = c_3 e^{-2t} + c_4 e^{4t}$

$y_0 = at+b$
$y' = a$

$-2a-8at-8b=-8t-2$
$a=1$
$-2-8b=-2 \quad b=0$

$y_0 = t$

$y_{GG} = c_3 e^{-2t} + c_4 . e^{4t} + t$

$x_{GG} = c_1 e^{-2t} + c_2 e^{4t} - t + 1$

$2\frac{dx}{dt} + \frac{dy}{dt} + x + 5y = 4t$

$2\left[-2c_1 e^{-2t} + 4c_2 e^{4t} - \cancel{1}\right] - 2c_3 e^{-2t} + 4c_4 e^{4t} + \cancel{1} +$
$+ c_1 e^{-2t} + c_2 e^{4t} - \cancel{t} + \cancel{1} + 5c_3 e^{-2t} + 5c_4 e^{4t} + \cancel{5t} = \cancel{4t}$

$-3c_1 e^{-2t} + 9c_2 e^{4t} + 3c_3 e^{-2t} + 9c_4 e^{4t} = 0$

$3(c_3-c_1)e^{-2t} + 9(c_2+c_4)e^{4t} = 0$

$c_1 = c_3 \qquad c_4 = -c_2$ 1

$y_{GG} = c_1 e^{-2t} - c_2 e^{4t} + t$ 1