2017 _ Bütünleme

(1) $\frac{dx}{dt} + \frac{dy}{dt} - 2x - 4y = e^{t}$

(2) $\frac{dx}{dt} + \frac{dy}{dt} - y = e^{4t}$

d.d.s. çöz. operatör yönt. ile çöz.

$(D-2)x + (D-4)y = e^{t}$

$Dx + (D-1)y = e^{4t}$

$\Delta = \begin{vmatrix} D-2 & D-4 \\ D & D-1 \end{vmatrix} = D^2-3D+2-D^2+4D = \underline{D+2}$

(keyfi sbt olmalı)

$\Delta x = \begin{vmatrix} e^{t} & D-4 \\ e^{4t} & D-1 \end{vmatrix} = (D-1)e^{t} - (D-4)e^{4t} = \cancel{e^{t}} - \cancel{e^{t}} - \cancel{4e^{4t}} + \cancel{4e^{4t}} = 0$

$(D+2)x = 0 \Rightarrow x' + 2x = 0 \Rightarrow \frac{dx}{dt} = -2x \Rightarrow$

$\frac{dx}{x} = -2dt \Rightarrow \ln x = -2t + \ln c_1 \Rightarrow \boxed{x = c_1 e^{-2t}}$

2)den: $-2c_1e^{-2t} + y' - y = e^{4t}$

$y' - y = 2c_1e^{-2t} + e^{4t}$ l.d.d.

$\lambda = e^{-\int dt} = e^{-t} \Rightarrow e^{-t}y' - e^{-t}y = 2c_1e^{-3t} + e^{3t} \Rightarrow$

$\frac{d}{dt}(e^{-t}y) = 2c_1e^{-3t} + e^{3t} \Rightarrow$

$e^{-t}\cdot y = -\frac{2}{3}c_1e^{-3t} + \frac{1}{3}e^{3t} + c_2$

$y = -\frac{2}{3}c_1e^{-2t} + \frac{1}{3}e^{4t} + c_2e^{t}$

(1)den $\cancel{-2c_1e^{-2t}} + \cancel{\frac{4}{3}c_1e^{-2t}} + \cancel{\frac{4}{3}e^{4t}} + c_2e^{t} - \cancel{2c_1e^{-2t}} + \cancel{\frac{8}{3}c_1e^{-2t}} - \cancel{\frac{4}{3}e^{4t}} - 4c_2e^{t} = e^{t}$

$-3c_2 = 1 \Rightarrow c_2 = -\frac{1}{3}$ //

1) $y'' + 16y = 5\sin x \quad y(0) = y'(0) = 0$ başl. değ. pr. Lapl. dön. kull. çöz.

$L[y''] + 16L[y] = 5L[\sin x]$

$s^2Y(s) - s\cancel{y(0)} - \cancel{y'(0)} + 16Y(s) = 5\cdot\frac{1}{s^2+1}$

$Y(s) = \frac{5}{(s^2+1)(s^2+16)}$

$y(x) = L^{-1}[Y(s)] = 5L^{-1}\left[\frac{1}{(s^2+1)(s^2+16)}\right]$

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(*) $\frac{1}{(s^2+1)(s^2+16)} = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+16}$

$1 = As^3 + 16As + Bs^2 + 16B + Cs^3 + Cs + Ds^2 + D$

$Y(s) = \frac{5}{(s^2+1)(s^2+16)}$

$A + C = 0$

$B + D = 0$

$16A + C = 0$

$16B + D = 1$

$A = 0 \Rightarrow C = 0$

$15B = 1 \quad B = 1/15 \Rightarrow D = -1/15$

$16B + D = 5$, $D = -\frac{1}{3}$

$A = 0, C = 0, B = 1/3$

$y(x) = \frac{1}{3}L^{-1}\left[\frac{1}{s^2+1}\right] - \frac{1}{3}L^{-1}\left[\frac{1}{s^2+16}\right]$

$= \frac{1}{3}\sin x - \frac{1}{12}\sin 4x$

$\frac{A}{s^2+1} + \frac{B}{s^2+16}$

2) $(x^2+1)y'' + xy' + 2xy = 0$ dif. denk.nin $x=0$ civ. kuv. seri çöz. bul.

$$y=\sum_{n=0}^{\infty} a_n x^n$$

$$y'=\sum_{n=1}^{\infty} a_n n x^{n-1} \quad ; \quad y''=\sum_{n=2}^{\infty} a_n n(n-1) x^{n-2}$$

$$(x^2+1)\sum_{n=2}^{\infty} a_n n(n-1)x^{n-2} + x\sum_{n=1}^{\infty} a_n n x^{n-1} + 2x\sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=2+2}^{\infty} a_n n(n-1)x^{n} + \sum_{n=2}^{\infty} a_n n(n-1)x^{n-2} + \sum_{n=1+2}^{\infty} a_n n x^{n} + \sum_{n=0+3}^{\infty} 2a_n x^{n+1} = 0$$

$$\sum_{n=4}^{\infty} a_{n-2}(n-2)(n-3)x^{n-2} + \sum_{n=2}^{\infty} a_n n(n-1)x^{n-2} + \sum_{n=3}^{\infty} a_{n-2}(n-2)x^{n-2} + \sum_{n=3}^{\infty} 2a_{n-3}x^{n-2} = 0$$

$$2a_2 + 6a_3x + a_1x + 2a_0x + \sum_{n=4}^{\infty}\left[a_{n-2}((n-2)(n-3+1)) + a_n n(n-1) + 2a_{n-3}\right]x^{n-2} = 0$$

$2a_2=0$ , $2a_0+a_1+6a_3=0$ , $a_3=-\frac{2a_0-a_1}{6}$ ; $n\geq 4$ $\quad a_n = \frac{-(n-2)^2}{n(n-1)}a_{n-2} - \frac{2}{n(n-1)}a_{n-3}$

$n=4$ için $a_4 = \frac{-4}{4.3}\cancel{a_2} - \frac{2}{4.3}a_1 \Rightarrow a_4 = -\frac{1}{6}a_1$

$n=5$ için $a_5 = \frac{-9}{5.4}a_3 - \frac{2}{5.4}\cancel{a_2} \Rightarrow a_5 = \frac{-9}{5.4}a_3$

$$y = a_0 + a_1x + \cancel{a_2}x^2 + a_3x^3 + \dots$$

$$= a_0 + a_1x + \left[-\frac{a_0}{3} - \frac{a_1}{6}\right]x^3 - \frac{1}{6}a_1x^4 - \frac{9}{20}\left(-\frac{a_0}{3} - \frac{a_1}{6}\right)x^5 \dots$$

$$= a_0\left[1 - \frac{x^3}{3} + \frac{3}{20}x^5 \dots\right] + a_1\left[x - \frac{1}{6}x^3 - \frac{1}{3}x^4 + \frac{3}{40}x^5 \dots\right]$$

3) $(3x-y-9)y' = 10-2x+2y$ dif. denk. gen. çöz.

$$(2x-2y-10)dx + (3x-y-9)dy = 0 \qquad \Delta = \begin{vmatrix} 2 & -2 \\ 3 & -1 \end{vmatrix} = 4 \neq 0$$

Homojen Hale Get. Dif. Denk.

$x = X+h$, $y = Y+k$ $\quad (2X-2Y+\underbrace{2h-2k-10}_{0})dX + (3X-Y+\underbrace{3h-k-9}_{0})dY$

$2h-2k=10$, $-2/$ $3h-k=9$ $\quad -4h=-8 \Rightarrow h=2 \Rightarrow k=-3 \Rightarrow x=X+2$, $y=Y-3$

$$(2X-2Y)dX + (3X-Y)dY = 0$$

$$\left(2-2\frac{Y}{X}\right)dX + \left(3-\frac{Y}{X}\right)dY = 0 \quad \text{homojen d.d.}$$

$\frac{Y}{X} = u$ $\quad dY = udX + Xdu$

$$(2-2u)dX + (3-u)(udX + Xdu) = 0$$

$$(2-2u+3u-u^2)dX + (3-u)X\,du = 0$$

$$\frac{dX}{X} + \frac{u-3}{u^2-u-2}du = 0 \qquad \frac{u-3}{u^2-u-2} = \frac{u-3}{(u-2)(u+1)} = \frac{A}{u-2} + \frac{B}{u+1}$$

$A+B=1$, $A-2B=-3$ ; $B=4/3$, $A=-1/3$

$$\int\frac{dX}{X} + \int\left(\frac{-1/3}{u-2} + \frac{4/3}{u+1}\right)du = \int 0$$

$$\ln X + \frac{1}{3}\left[-\ln|u-2| + 4\ln|u+1|\right] = \ln c \qquad \left(u=\frac{Y}{X},\ X=x-2,\ Y=y+3\right)$$

$$X^3 \cdot \frac{(u+1)^4}{(u-2)} = C \qquad Y+X=x$$

$$X^3\frac{\left(\frac{Y}{X}+1\right)^4}{\left(\frac{Y}{X}-2\right)} = C \Rightarrow \frac{(Y+X)^4}{(Y-2X)} = C \Rightarrow \frac{(x+y+1)^4}{y-2x+7} = C$$

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4.) $\begin{cases} \dfrac{dx}{dt}+\dfrac{dy}{dt}-2x-4y=e^{t} \\ \dfrac{dx}{dt}+\dfrac{dy}{dt}-y=e^{4t} \end{cases}$ diferansiyel denklem sisteminin genel çözümünü **determinant** yöntemini kullanarak çözünüz.

$\frac{d}{dt}=D$

$Dx+Dy-2x-4y=e^{t}$

$Dx+Dy-y=e^{4t}$

$(D-2)x+(D-4)y=e^{t}$

$Dx+(D-1)y=e^{4t}$

$\boxed{\Delta=D+2}$

$$\frac{\begin{vmatrix} D-2 & e^{t} \\ D & e^{4t} \end{vmatrix}}{\begin{vmatrix} D-2 & D-4 \\ D & D-1 \end{vmatrix}}=y$$

1 tane olmalı

$$y=\frac{4e^{4t}-2e^{4t}-e^{t}}{D^2-3D+2-D^2+4D}=\frac{2e^{4t}-e^{t}}{D+2}$$

$(D+2)y=2e^{4t}-e^{t}$

$y'+2y=2e^{4t}-e^{t}$

$r=-2$

$u=c_1e^{-2t}$

$v_1=ke^{4t}$

$v_1'=4ke^{4t}$

$4ke^{4t}+2ke^{4t}=2e^{4t}$

$k=1/3$

$\boxed{v_1=\frac{e^{4t}}{3}}$

$\boxed{y=c_1e^{-2t}+\frac{e^{4t}}{3}-\frac{e^{t}}{3}}$

$v_2=be^{t}$

$v_2'=be^{t}$

$be^{t}+2be^{t}=-e^{t}$

$3b=-1$

$b=-1/3$

$\boxed{v_2=\frac{-e^{t}}{3}}$

$$x=\frac{\begin{vmatrix} e^{t} & D-4 \\ e^{4t} & D-1 \end{vmatrix}}{\begin{vmatrix} D-2 & D-4 \\ D & D-1 \end{vmatrix}}=\frac{e^{t}-e^{t}-4e^{4t}+4e^{4t}}{D+2}$$

$x(D+2)=0$

$x'+2x=0$

$r=-2$

$\boxed{x=c_2e^{-2t}}$

$-2c_2e^{-2t}-2c_1e^{-2t}+\frac{4}{3}e^{4t}-\cancel{\frac{e^{t}}{3}}-c_1e^{-2t}-\cancel{\frac{e^{4t}}{3}}+\cancel{\frac{e^{t}}{3}}=e^{4t}$

$-2c_2-3c_1=0$

$\boxed{-2c_2=3c_1}$

$\boxed{c_2=-\frac{3}{2}c_1}$

$\boxed{x=-\frac{3}{2}c_1e^{-2t}}$