4) $(x-1)y'' + y' = (x-1)^2$ G.G ?

$y' = p \quad y'' = p'$

$(x-1)p' + p = (x-1)^2$

$p' + \frac{1}{x-1}p = (x-1)$ lineer d.d

$\lambda(x) = e^{\int \frac{1}{x-1}dx} = e^{\ln(x-1)} = (x-1)$

Σ:

$(x-1)p' + p = (x-1)^2$

$\frac{d}{dx}[(x-1)p] = (x-1)^2$

$\int d[(x-1)p] = \int (x-1)^2 dx$

$(x-1)p = \frac{(x-1)^3}{3} + c_1$

(✱)

$p = y' = \frac{(x-1)^2}{3} + \frac{c_1}{(x-1)}$

$y = \int \left[\frac{(x-1)^2}{3} + \frac{c_1}{(x-1)}\right]dx$

$= \frac{(x-1)^3}{9} + c_1 \ln|x-1| + c_2$

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veya (✱) **2.yol:**

$P = \frac{1}{x-1}\left[\int (x-1)(x-1)dx + c_1\right]$

$P = \frac{1}{x-1}\left[\frac{(x-1)^3}{3} + c_1\right] = \frac{(x-1)^2}{3} + \frac{c_1}{x-1}$

2017 2.Vize

1) $y'' - 3y' + 2y = \frac{e^{3x}}{1+e^x}$ Par. Değ. Yönt.

$r^2 - 3r + 2 = 0 \quad (r-1)(r-2) = 0 \quad r_1 = 1 \quad r_2 = 2$

$y_h = c_1 e^x + c_2 e^{2x}$

$y_{GG} = c_1(x).e^x + c_2(x).e^{2x}$

$-1 / \quad c_1'.e^x + c_2'.e^{2x} = 0$

$c_1' e^x + c_2'.2e^{2x} = \frac{e^{3x}}{1+e^x}$

+

$c_2'.e^{2x} = \frac{e^{3x}}{1+e^x} \Rightarrow c_2' = \frac{e^x}{1+e^x}$

$c_1'.e^x + \frac{e^x}{1+e^x}.e^{2x} = 0 \Rightarrow c_1' = -\frac{e^{2x}}{1+e^x}$

25

$c_1(x) = -\int \frac{e^{2x}}{1+e^x}dx = -\int \frac{(u-1).du}{u} = -\int\left(1-\frac{1}{u}\right)du = -u + \ln|u| + k_1$

$1+e^x = u \quad e^x dx = du$

$c_1(x) = -(1+e^x) + \ln(1+e^x) + k_1$

25

$c_2(x) = \int \frac{e^x}{1+e^x}dx = \int \frac{du}{u} = \ln|u| + k_2$

$1+e^x = u$
$e^x dx = du$

$c_2(x) = \ln(1+e^x) + k_2$

$y_{GG} = [-(1+e^x) + \ln(1+e^x) + k_1]e^x + [\ln(1+e^x) + k_2]e^{2x}$

2.) $y''' - y'' + y' - y = e^x + 2\cos x - 4\sin x$ Belirsiz kats. yönt.

$$\left\{\begin{array}{lll} r^3 - r^2 + r - 1 = 0 & r^2(r-1) + (r-1) = 0 & (r-1)(r^2+1) = 0 \\ r_1 = 1 & r_{2,3} = \pm i & \end{array}\right.$$

$y_h = c_1 e^x + c_2 \cos x + c_3 \sin x$

$$\begin{array}{rl} -1/ & y_{\ddot{o}_1} = A\cancel{x}e^x \\ & y' = A(1+\cancel{x})e^x \\ -1/ & y'' = A(2+\cancel{x})e^x \\ + & y''' = A(3+\cancel{x})e^x \end{array}$$

$2Ae^x = e^x \Rightarrow A = \frac{1}{2} \Rightarrow y_{\ddot{o}_1} = \frac{1}{2}xe^x$

$$\begin{array}{rl} -1/ & y_{\ddot{o}_2} = x(a\cancel{\cos}x + b\cancel{\sin}x) \\ & y' = (a\cos x + b\sin x) + x(-a\cancel{\sin}x + b\cancel{\cos}x) \\ 1/ & y'' = 2(-a\sin x + b\cos x) + x(-a\cancel{\cos}x - b\cancel{\sin}x) \\ + & y''' = 3(-a\cos x - b\sin x) + x(+a\cancel{\sin}x - b\cancel{\cos}x) \end{array}$$

$(-2a-2b)\cos x + (-2b+2a)\sin x = 2\cos x - 4\sin x$

$$\left.\begin{array}{l} -\cancel{2a} - 2b = 2 \\ -2b + \cancel{2a} = -4 \end{array}\right\} \quad -4b = -2 \quad b = \frac{1}{2} \Rightarrow a = -\frac{3}{2}$$

$y_{\ddot{o}_2} = x\left(-\frac{3}{2}\cos x + \frac{1}{2}\sin x\right)$

$y_{G.Ç.} = c_1e^x + c_2\cos x + c_3\sin x + \frac{1}{2}xe^x + x\left(-\frac{3}{2}\cos x + \frac{1}{2}\sin x\right)$

8 − 12 − 13 − 7 − 57 − 20 − 18 − 1

3.) $x^2y'' - 2xy' + 2y = 4x^3 . \ln x$ G.Ç. = ?

$$\left.\begin{array}{l} x = e^t \\ y' = e^{-t}Dy \\ y'' = e^{-2t}D(D-1)y \end{array}\right\}$$

$\cancel{e^{2t}}\,\cancel{e^{-2t}}D(D-1)y - 2\cancel{e^{t}}.\cancel{e^{-t}}Dy + 2y = 4e^{3t}.t$

$D^2y - Dy - 2Dy + 2y = 4te^{3t}$

$\boxed{y'' - 3y' + 2y = 4te^{3t}}$

$r^2 - 3r + 2 = (r-2)(r-1) = 0 \quad r_1 = 1 \quad r_2 = 2$

$y_h = c_1e^t + c_2e^{2t}$

$$\begin{array}{rl} +2/ & y_{\ddot{o}} = (At+B).e^{3t} \\ -3/ & y' = (\cancel{A} + 3\cancel{A}t + 3\cancel{B})e^{3t} \\ + & y'' = (\cancel{3A} + 3A + 9\cancel{A}t + 9\cancel{B})e^{3t} \end{array}$$

$(2At + 2B + 3A)e^{3t} = 4te^{3t}$

$2A = 4 \quad A = 2, \quad 2B + 3A = 0 \quad B = -3$

$y_{\ddot{o}} = (2t-3)e^{3t}$

$y_{GÇ} = c_1e^t + c_2e^{2t} + (2t-3)e^{3t}$

$= c_1x + c_2x^2 + (2\ln x - 3)x^3$

3.yol: $x^2y'' - 2xy' + 2y = 4x^3\ln x$

$$\left.\begin{aligned} &y = x^r,\ y' = r.x^{r-1},\ y'' = r(r-1)x^{r-2} \end{aligned}\right\}$$

$$\left.\begin{aligned} &x^2.r(r-1)x^{r-2} - 2xr\,x^{r-1} + 2x^r = 4x^3\ln x \\ &(r^2-r)x^r - 2rx^r + 2x^r = 4x^3\ln x \\ &(r^2-3r+2)x^r = 4x^3\ln x \end{aligned}\right\}$$

K.D. $r^2-3r+2=0 \Rightarrow r_1=2,\ r_2=1$ $\quad y_H = c_1x + c_2x^2$

$y_1 = x$ $\quad y_p = v_1y_1 + v_2y_2$

$y_2 = x^2$

$$v_1 = \int \frac{-f(x)y_2}{a.W}dx \quad , \quad v_2 = \int \frac{f(x)y_1}{a.W}dx$$

$$W = \begin{vmatrix} x & x^2 \\ 1 & 2x \end{vmatrix} = x^2 \qquad a = x^2 \quad , \quad f(x) = 4x^3\ln x$$

$$v_1 = \int \frac{-4x^3\ln x \,.\, x^2}{x^2.x^2}dx = -4\int x\ln x\,dx = -4\left[\frac{x^2}{2}\ln x - \frac{x^2}{4}\right]$$

$$v_2 = \int \frac{4x^3\ln x \,.\, x}{x^2.x^2}dx = 4\int \ln x\,dx = 4\left[x\ln x - x\right]$$

$$y_p = x.\left[-2x^2\ln x + x^2\right] + x^2\left[4x\ln x - 4x\right]$$

$$y = y_H + y_p = c_1x + c_2x^2 - 2x^3\ln x + x^3 + 4x^3\ln x - 4x^3$$

$$y = c_1x + c_2x^2 + x^3\left[2\ln x - 3\right]$$

4. yol

$y_2 = z(t)e^{3t}$ ☺

$y_2' = z'e^{3t} + 3ze^{3t} = (z' + 3z)e^{3t}$ ☺

$y_2'' = (z'' + 3z')e^{3t} + (3z' + 9z)e^{3t}$

$= (z'' + 6z' + 9z)e^{3t}$ ☺

$(z'' + 6z' + 9z - 3z' - 9z + 2z)\cancel{e^{3t}} = 4t\,\cancel{e^{3t}}$

☺ $z'' + 3z' + 2z = 4t$ $\quad (r^2 + 3r + 2 = 0 \quad (r+1)(r+2) = 0)$ $\quad r_{1,2}$

$z_2 = at + b$

$z_2' = a$

$z_2'' = 0$

$0 + 3a + 2at + 2b = 4t$

$2a = 4$ $\quad 3a + 2b = 0$

$\boxed{a = 2}$ ☺ $\quad \boxed{b = -3}$ C

$z_2 = 2t - 3$

$\boxed{y_2 = (2t - 3)e^{3t}}$ C

$$C_1' e^t + C_2' e^{2t} = 0$$
$$-/ \; C_1' e^t + 2C_2' e^{2t} = 4te^{3t}$$

$u = t$, $du = dt$; $dv = e^t dt$, $v = e^t dt$; $dv = e^{2t} dt$, $v = \frac{e^{2t}}{2}$

$$\cancel{+}C_2' e^{2t} = \cancel{+}4te^{3t}$$
$$\boxed{C_2' = 4t\cdot e^t} \Rightarrow 4[t\cdot e^t - e^t] + k_2$$
$$C_1' \cancel{e^t} = -4t\cancel{e^t}\cdot e^{2t} \qquad \boxed{C_1' = -4te^{2t}} \Rightarrow -4\left[\frac{te^{2t}}{2} - \frac{e^{2t}}{4}\right] + k_1$$

$$y = c_1 e^t + c_2 e^{2t}$$
$$y = -2te^{3t} + \underline{\underline{e^{3t}}} + k_1 e^t + 4te^{3t} \underline{\underline{-4e^{3t}}} + k_2 e^{2t}$$
$$y = k_1 e^t + k_2 e^{2t} - 3e^{3t} + 2te^{3t}$$
$$y = \;''\quad '' + e^{3t}[2t - 3]$$
$$y = k_1 x + k_2 x^2 + x^3[2\ln x - 3]$$

5. yol

$$\frac{1}{x} / \; x^2 y'' - 2xy' + 2y = 4x^3 \ln x$$
$$xy'' - 2y' + 2\frac{y}{x} = 4x^2 \ln x$$

$$\begin{cases} y = ux \\ y' = u'\cdot x + u \\ y'' = u''\cdot x + u' + u' = u''x + 2u' \end{cases}$$

$$x\cdot[u''x + 2u') - 2(u'x + u) + 2u = 4x^2 \ln x$$
$$u''x^2 + 2\cancel{u'}\cdot x - \cancel{2u'}x - \cancel{2u} + \cancel{2u} = 4x^2 \ln x$$
$$u''\cancel{x^2} = 4\cancel{x^2}\ln x \Rightarrow u'' = 4\ln x \Rightarrow u' = 4\int \ln x\,dx$$
$$u' = 4[x\ln x - x] + k_1 \Rightarrow u = 4\int (x\ln x - x + k_1)dx$$
$$\Rightarrow u = 4\int \underbrace{\ln x}_{u}\cdot \underbrace{x\,dx}_{dv} - \frac{4x^2}{2} + 4k_1 x + k_2$$
$$\Rightarrow u = 4\cdot\left[\frac{x^2}{2}\cdot\ln x - \int \frac{x^{\cancel{2}}}{2}\cdot\frac{1}{\cancel{x}}dx\right] - 2x^2 + 4k_1 x + k_2$$
$$\Rightarrow u = 4\left[\frac{x^2}{2}\ln x - \frac{1}{2}\cdot\frac{x^2}{2}\right] - 2x^2 + 4k_1 x + k_2$$
$$\Rightarrow u = 2x^2 \ln x - x^2 - 2x^2 + 4k_1 x + k_2$$
$$\Rightarrow u = 2x^2 \ln x - 3x^2 + 4k_1 x + k_2$$
$$\Rightarrow \frac{y}{x} = x^2\cdot[2\ln x - 3] + \underbrace{4k_1}x + k_2$$
$$\Rightarrow \boxed{y = x^3\cdot[2\ln x - 3] + c_1 x^2 + c_2 x}$$

2 pt

$$x^2y'' - 2xy' + 2y = 4x^3\ln x$$

$$\left.y = x^r,\ y' = r x^{r-1},\ y'' = r(r-1)x^{r-2}\right\}$$

$$\left.\begin{aligned}
&x^2\cdot r(r-1)x^{r-2} - 2x\, r x^{r-1} + 2x^r = 4x^3\ln x\\
&(r^2-r)x^r - 2r x^r + 2x^r = 4x^3\ln x\\
&(\underbrace{r^2-3r+2}) x^r = 4x^3\ln x
\end{aligned}\right\}$$

$$\left.\begin{aligned}
\text{K.D:}\quad & r^2-3r+2=0\\
&(r-2)(r-1)=0\\
&r_1=2 \quad r_2=1 \Rightarrow y_h = c_1x + c_2x^2
\end{aligned}\right\}$$

$$c_1'x + c_2'x^2 = 0$$

$$-x\,/\ c_1' + 2c_2'x = \frac{4x^3\ln x}{x^2\,(=a_0)}$$

$$-c_2'x^2 = -4x^2\ln x$$

$$c_2' = 4\ln x \Rightarrow c_2 = 4(x\ln x - x) + k_2$$

$$c_2'x^2 = 4x^2\ln x \Rightarrow c_1'x = -4x^2\ln x$$

$$c_1' = -4x\ln x$$

$\ln x = u$

$\frac{dx}{x} = du$

$x\,dx = dv$

$\frac{x^2}{2} = v$

$$\begin{aligned}
c_1 &= -4\int x\ln x\,dx + k_1\\
&= -4\left[\frac{x^2}{2}\ln x - \frac{x^2}{4}\right] + k_1\\
&= -2x^2\ln x + x^2 + k_1
\end{aligned}$$

$$\left.\begin{aligned}
y &= x\left[-2x^2\ln x + x^2 + k_1\right] + \left[4(x\ln x - x) + k_2\right]x^2\\
&= -2x^3\ln x + x^3 + k_1x + 4x^3\ln x - 4x^3 + k_2x^2\\
&= k_1x + k_2x^2 + x^3\left[2\ln x - 3\right]
\end{aligned}\right\}$$